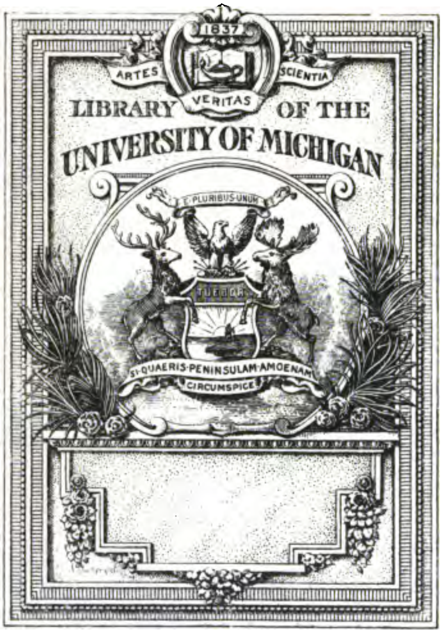
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LIGHT

LIGHT

BY

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PREFACE.

This work is, like the little treatise on Heat which I have just published, based upon the system which, after many years' experience, I have adopted in my ordinary Lectures. One of my chief reasons for bringing out such volumes has been the impossibility of treating with adequate detail, in a single Session, each branch of Experimental Physics. I have always given, with the requisite experimental illustrations, the fundamental phenomena and laws of each branch — further detail being necessarily confined to two or three of them, which are varied from Session to Session. An Elementary Treatise like this will therefore supply, for the Student's private reading, what time does not permit his obtaining in the Lecture-room, in those Sessions in which Light is made to hold a less prominent place than Heat, Sound, or Electricity.

The book is thus not designed for those who intend to make a special study either of theoretical or of experimental Optics, but for ordinary students who wish to acquire familiarity with the elements of the subject.

It is in no sense a mere reprint of the article "Light" in the new edition of the *Encyclopædia Britannica*. The plan of that work required that the subject should be treated by instalments, under very different heads; and my article was necessarily limited to a simple sketch, whose main object was to co-ordinate these detached portions, some of which are not yet even written. Thus, for instance, a mere mention was made of Caustics, Halos, Fresnel's Wave-Surface, etc.; while such subjects as Focal Lines, Glories, the effect of Prisms of large angle, and the physical basis of Spectrum Analysis, were entirely omitted.

This work was, unfortunately, all in type before Prof. Stokes' *Burnett Lectures* appeared, so that I have not been able to avail myself of any part of the remarkable cumulative argument in favour of the Undulatory Theory which is the main object of these Lectures.

As the book has been prepared under the pressure of a very busy session, and as I have had but slight assistance in correcting the press, I cannot hope that it will be found altogether free from error.

P. G. TAIT.

COLLEGE, EDINBURGH,
March 1, 1884.

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LIGHT.

CHAPTER I.

INTRODUCTORY AND HISTORICAL.

1. Sound may be defined as any effect on the sense of hearing, and in the same way Light may be defined as any effect on the sense of sight. This is the purely subjective use of the terms. But both terms are quite as frequently used in the objective as in the subjective sense. Thus, as Sound may be defined in terms of the motion of the air in the cavity of the external ear, mechanically affecting the tympanum, so Light may be defined by the mechanical effect produced upon the extension of the optic nerve which forms the sensitive surface of the retina.

In treating of Light it will be convenient to use the term in a sort of mixed sense, at least until we come to discuss the different theories which have been devised to account for the propagation of the agent which causes vision. Then we shall have to use the term entirely in the objective sense. On the other hand, in Physiological Optics (about which a few words will be said in next chapter) we are concerned chiefly with the subjective sense of the term.

2. The present work is intended to give a general sketch of the subject of Optics, so far as it can be treated by the help of elementary mathematics, but with sufficient detail to show the connection of its various branches.

Many of these, such as the construction and use of Telescopes, or of Microscopes, and the processes of Spectrum Analysis, in which the true theory has been at least partially ascertained, have been developed to such an extent that the full treatment of any one of them, even by elementary methods, would require at least one large volume. Others, which promise to become of at least equal importance, especially in the information which we hope to derive from them as to the nature of matter and of the ether, are still in that exceedingly fragmentary and chaotic state which indicates the absence of an adequate theory to enable us to marshal the facts.

3. Comparatively recent discoveries have shown us that, just as there are sounds too grave or too shrill to be perceived by the human ear, so there are rays of light whose vibrations are too slow or too rapid to affect the human eye (see Chapter XVI).

What was, not very long ago, almost universally called *Radiant Heat*, which we discover by the sense of touch, and measure by the thermo-electric pile and galvanometer, is now known to be merely the graver forms of luminous vibrations.

And the so-called *Actinic Rays*, which were discovered by their special activity in connection with the earlier photographic processes, but which can now (by the aid of fluorescent substances) be changed into visible rays, are merely vibrations too rapid to affect the eyes.

Light, in fact, now takes its place alongside of electric phenomena, as but one of the forms of energy associated with that wonderful kind of matter provisionally called the ether.

4. It is to sight that we are mainly indebted for our knowledge of external things. All our other senses together, except under very special conditions, do not furnish us with a tithe of the information we gain by a single glance. And sight is also that one of our senses which we are able most effectively and extensively to aid by the help of proper apparatus—not merely (as by spectacles, invented *circa* 1300) for the cure of natural defects, but (as by the telescope and microscope) for the examination of bodies either too distant or too minute to be studied by the unassisted eye.

5. It is very remarkable, under these circumstances, to find how slowly the human race has reached some even of the simplest facts of optics. We can easily understand how constant experience must have forced on men the conviction that light usually moves in straight lines—*i.e.* that we see an object in the direction in which it really lies. But how they could have believed for ages that objects are rendered visible by something projected from the eye itself—so that the organ of sight was supposed by the most enlightened of them to be analogous to the tentacula of insects, and sight itself a mere species of touch—is most puzzling. They seem not till about 350 B.C. to have even raised the question —If this is how we see, why cannot we see in the dark? or, more simply,—What is darkness? The former of

these questions appears to have been first put by Aristotle.

6. The nature and laws of *Reflection* were, of course, forced on the ancients by the images seen in still water; and the geometers of the Platonic school were well acquainted with these laws. To Hero of Alexandria we owe the important deduction from them that the course of a reflected ray is the shortest possible.

7. The general nature of *Refraction* also was known, with some of its special applications, such as, for instance, to the construction of burning-glasses and magnifiers. These were probably either spherical glass shells filled with water or balls of rock crystal.

8. In the first century of our era Cleomedes pointed out how a coin at the bottom of an empty cup, where the eye cannot see it, can be made visible by filling the cup with water; and he showed that, in a similar way, the air may render the sun visible to us while it is still below the horizon.

Shortly after this date Ptolemy (the celebrated astronomer) published his great work on *Optics*. He treats of vision, reflection, the theory of plane and of concave mirrors, and of refraction. He measured, with considerable accuracy, the angles of incidence and refraction, for rays passing from air into water and into glass, and from water into glass; it was not, however, till more than fifteen hundred years had passed that the true relation between these angles was discovered.

In addition to what has just been mentioned, the ancients' knowledge of optics was limited to a very superficial acquaintance with some of the properties of rainbows, halos, mirage, etc. But it was fragmentary in the extreme—though it far surpassed in amount as well as in accuracy their knowledge of the other branches of physical science.

9. It is not easy to understand the ideas of the ancients about *Colour*. That it is a property of a body —just as its density, its hardness, or its smell is a property—was probably held by them. But they also imagined that a body could communicate its colour to light; thus, for instance, the clouds were, by some of them, supposed to communicate their colours to the sunbeams which form a rainbow.

10. Our next glimpse of real progress dates from the end of the eleventh or the beginning of the twelth century, when Alhazen[1] wrote a treatise on optics in Arabic, which for five hundred years or more was in Europe the recognised authority on the subject. It was, in many parts, founded on the work of Ptolemy, but with considerable additions and improvements. Alhazen gives an anatomical description of the eye, and points out, fairly enough, how with two eyes we see only one image. But he also points out that we see each object, however small, by a *pencil* of diverging rays,—not (as the ancients imagined) by a single ray. Alhazen accounts for twilight, and shows how by it to measure the height of the atmosphere. He also gives the now generally received explanation of the curious fact that the sun and moon appear larger when rising or setting than when they are high in the heavens.

11. The further progress of the subject we need not now

[1] The proper name of this geometer is El-Hasan (or by other accounts Mohammed) ibn el-Hasan ibn el-Haitham, and it is as Ibn el-Haitham that he is commonly referred to.

trace. From the end of the sixteenth century that progress has been extremely rapid. The dates of the more important steps, and the names of their authors, will be given when we treat of these, in their turn, in the course of the work; and we will give them the additional interest of being presented in the authors' own words (if in English), or in a close paraphrase of them.

CHAPTER II.

PRELIMINARY STATEMENTS.

12. Before we commence a more rigorous treatment of the subject, it may be well to make a few preliminary statements as to the nature of *Vision* and the conditions for *distinct vision*. Properly speaking, these belong to Physiological Optics, a subject quite beyond the proper range of this work; but it is impossible to treat intelligibly any part of our subject without presupposing some, generally very slight, knowledge of other parts. And the few preliminary statements we have now to make are in no respect theoretical, while they are so simple that any one may at once test their truth for himself.

The reader may expect to feel, in this short chapter, the inevitable inconvenience which results from the intimate interdependence of the various parts of our subject. This is characteristic of all branches of science. No one part can be fully studied alone; each requires assistance from, or at least reference to, others; so that the problem of how and where to begin is one of the most difficult that an author has to face. The terms, however, which we must introduce (without explanation) in the present chapter are, with few exceptions, such as the reader may reasonably be supposed to have met with before. And, should he but imperfectly understand them at this stage, he will be referred back to the present use of them when the time for their full explanation has arrived.

13. Except in the case of a very abnormal eye (extremely short-sighted or long-sighted as the case may be) there is a distance from it—usually somewhere about 10 inches—at which if an object be placed, it is seen more distinctly than if placed at any other distance. Almost every one, perhaps without knowing it, habitually places at or about that distance from his eye an object which he wishes to examine carefully. When he places it at a smaller distance he becomes conscious of the *effort* required to see it distinctly. He has, in fact, to alter the form of the optical machinery of the eye, by a muscular effort, so that it may become capable of bringing to a focus, on the retina, rays more divergent than those for which the parts were in their unstrained state adapted. If the object be at a distance greater than 10 inches, he can still see it distinctly; but he cannot examine it with such detail as before, because he necessarily sees it under a smaller angle.

14. Hence we arrive at the conclusion that, for the maximum distinctness of vision, rays should fall on the eye diverging as if they came from a point about 10 inches distant. But for all ordinary eyes any divergence from double of this (*i.e.* divergence as if from a distance of 5 inches) to zero (*i.e.* parallel rays) is consistent with the possibility of distinct vision. Rays either more divergent than the former limit, or convergent, are unfit to produce distinct vision.

15. Hence every optical instrument, whatever be the

reflections or refractions to which light has been subjected in passing through it, must finally allow the light to escape either in parallel rays, or with a divergence within the above specified limits, if it is to be employed by an ordinary eye. The comparatively slight differences which exist among ordinary eyes are easily compensated by the rack-work, or screw adjustment, which is invariably attached to the eye-piece of a good telescope and to the body of a good microscope. Every motion of this rack-work alters the divergence of the rays as they finally escape from the instrument.

16. Any eye, however abnormal, if it be capable of producing distinct vision at all, has only to be furnished with suitable spectacles in order that it may behave exactly as does a normal eye.

[A peculiar defect of some eyes is that the curvature of the lens (or of the cornea?) is not the same in two sections at right angles to one another :—*i.e.* these bodies are not figures of revolution. This defect can, to a great extent, be corrected by spectacles which possess similar but opposite properties. A glance at the shading of figures (1) and (3), which follow, will enable the reader to discover whether or not his eyes have this peculiarity. It must necessarily be present if he cannot see distinctly, and simultaneously, the several vertical, horizontal, and inclined lines in the shaded parts of these figures.]

This statement, however, refers only to sharpness of definition, not in any degree to *colour*. The deficiency which causes *colour-blindness* cannot be supplied by any conceivable process. A definite part of the ordinary organ of vision is wanting (or inactive) in such cases—while the merely optical parts of the eye are usually in perfect order.

17. Another fact which must be stated here is that, to produce vision of a body in its natural position, the image on the retina, as seen from the back, must be inverted—not merely as regards up and down, but also as regards right and left. Thus, in the ordinary astronomical telescope, the image on the retina is not inverted, and we therefore see an inverted image.

18. A third is that our judgment of the relative distances of various objects, or of the parts of any one object, is formed mainly by the use of the two eyes simultaneously. One eye, kept still, can inform us only of relative distance in virtue of the greater or less effort to see distinctly (already spoken of). With both eyes, or with one eye moved from side to side, *parallax* comes in, and gives us the *stereoscopic* effect, as it is called. This power of judging distance is, of course, greater as the eyes are set more widely apart. There is, practically, no limit to the effective distance between the eyes when the proper instrumental methods (as with the telestereoscope) are employed.

When two pictures of the same object, represented side by side *on one plane* as the object would be seen by either eye respectively, are looked at (one by each eye), the impression produced on the mind is that of the *solid* figure of which they are pictures. If one type in a printed page has been, ever so slightly, displaced between two impressions, we can detect it in a moment by this process, for it appears to be above or below the plane in which the others lie. Similarly, a forged bank-note may be detected by comparing it with a genuine one.

19. It is also necessary to premise a few words about colour. The various homogeneous rays of the solar

spectrum have each a colour of its own which no refraction can modify. But what about the many colours which do not occur in the spectrum? To such a question as "What is *yellow?*" the answer is, "*Each particular kind of yellow may be any one of an infinite number of different combinations of homogeneous rays.*" And the same is true, in general, of all other colours.

Clerk-Maxwell found that a yellow, equivalent to that of the spectrum, can be obtained by mixing in proper proportions certain homogeneous red and green rays. This single example is sufficient to show that the colour-sense is of a very peculiar nature.

20. This question belongs wholly to Physiological Optics, and, as such, is outside the range of this work:—but for our present purpose it is only necessary to say that we have strong reasons for believing (after Wünsch and Young) that the normal eye has only *three colour-sensations*—a red, a green, and a violet—and that the apparent colour of any light which falls on it depends merely on the *relative intensities of the excitement* produced by the light on the three organs of sense corresponding to these sensations. This is true, however, only within certain limits of intensity: for extremely bright light, whatever be its real colour, seems to excite all the three sensations simultaneously, much as white light does; and with very feeble light (as, for instance, that of an ordinary aurora or of a lunar rainbow) we are sometimes scarcely conscious of colours.

21. In *colour-blindness* one or more of these organs of sense is wanting, or imperfect. The most common form, Daltonism, depends on the absence of the red sense. Great additions to our knowledge of this subject, if only

in confirmation of results already deduced from theory, have been obtained in the last few years by Holmgren;[1] who has experimented on two persons, each of whom was found to have one colour-blind eye, the other being nearly normal. In this way was obtained, what could otherwise have been matter of conjecture only, a description of colour-blind vision in terms of (at least approximately) normal vision.

22. Finally, the sensation of sight is not limited to the duration of the mechanical action on the eye. It is known that we do not see a sudden flash (an electric spark, for instance) until a measurable, though very short, period has elapsed. This depends on the rate at which an excitation is propagated along the optic nerve. But the familiar experiment of whirling a red-hot stick in a dark room shows that the sensation of sight lasts for a short period after the mechanical action which produced it has ceased. This period is probably different for different eyes, and for different amounts of excitement even in the same eye. (If the light be very intense the effect lasts much longer, but completely changes its character.) For our present purpose it may be assumed that the duration is somewhere about $\frac{1}{7}$th of a second. Thus, if the end of the red-hot stick describes a circle once in $\frac{1}{7}$th of a second, we see the complete circle; if in a longer period, we only see at once such a part of it as was described in $\frac{1}{7}$th of a second.

Connected with this is the remarkable result obtained experimentally by Swan,[2] that the amount of sensation is, for flashes of short duration, directly proportional,

[1] *Proc. Roy. Soc.*, Jan. 1881.
[2] *Trans. Roy. Soc. Edin.*, 1849, 1861.

not only to the brightness of the flash, but also to its duration. A flash which lasts for $\frac{1}{10}$th of a second produces the full effect on the eye; but an electric spark, such as a flash of lightning, which certainly does not endure for more than $\frac{1}{1000000}$th of a second, produces at most only $\frac{1}{100000}$th of the effect it would produce if it lasted $\frac{1}{10}$th of a second.

23. On this short, though essentially finite, duration of visual impressions depends the action of the *thaumatrope*, the *wheel of life*, etc. In the former we are presented with views of two different objects, or different parts of the same object, in rapid succession; and we combine them into one picture. In the second, a succession of views of an object in different positions or forms is presented to the eye, each for a brief interval. The result is that we fancy we see one and the same object going through a species of *continuous* motion, or of change of form, which would present it to the eye in these successive positions or forms. Thus, a tadpole may be represented as wriggling about, or as developing continuously into a frog, etc. Recent improvements in photography have made it possible to take successive instantaneous pictures, of a horse in the act of trotting, at intervals sufficiently short to enable us, by the use of the wheel of life, to reproduce fully the action of the horse's legs.

CHAPTER III.

SOURCES OF LIGHT.

24. THIS subject is really, from the theoretical point of view, the question of the origin of *Radiation;* while from the practical point of view it expressly includes the whole subject of candles, lamps, electric lighting, etc. The former of these is a legitimate part of our work, and will be treated with some little detail farther on; the latter is altogether beyond our province. But the general theory must be learned by the student from some of the recent really scientific works upon Heat, Thermodynamics, or Energy generally. For our present purpose a very brief summary of the question will suffice; as we do not for the moment require to investigate the *process* by which, in any case, the light is produced.

25. The main source of light is *Incandescence.* (It is usually understood that to be incandescent a body must be at a high temperature.) This may be due to any of a number of causes, such as the following:—

(*a*) *The Potential Energy of Gravitation of Scattered Fragments of Matter.*—When these fall together, as in the formation of the sun and stars, heat enough is generated by impact to render the whole vividly incandescent. It is probable that the light of nebulæ, and the *proper* light of comets, is due to this cause. The proximate cause, in all these cases, is the kinetic energy of the fragments before impact. To this class, therefore, can be reduced the light given out when a target is struck by a cannon shot.

(*b*) *The Kinetic Energy of Current Electricity or of an Electric Discharge.*—Here we have lightning, the electric light (whether it be the arc-light or the incandescent light), and probably also the light of the aurora.

(*c*) *The Potential Energy of Chemical Affinity.*—The lime-light, gas-light, candle and lamp-light, fire-light, the magnesium light, etc.; also phosphorus, dead fish (?), etc., glowing in the dark.

(*d*) *Friction*, as in the trains of sparks from a grind-stone or brake; though here, in general, chemical affinity also has a share.

(*e*) *Sudden great Compression of a Gas*, as of air by meteoric stones and falling stars.

26. Another very curious source, not (so far as is known) reducible to incandescence, is *the giving out* (*usually in an altered form*) *of light previously absorbed:*—fluorescence, phosphorescence, luminous paints, etc. Sometimes a body is rendered phosphorescent by comparatively moderate heating, with the exclusion of all visible light.

27. A third source is *physiological*:—fire-flies, glow-worms, *Medusæ*, dead fish (?), etc. These, from their very nature, cannot be further treated here.

28. Any not black and not transparent body, exposed to any of these sources of light, becomes in its turn what may for our purpose also be treated as a source.

29. As will be shown when we deal with Radiation, the only bodies which, when incandescent, give *every* constituent of white light, are bodies which are black in the sense of absorbing each and every ray which falls upon them. Such bodies are not *necessarily* solids—though the best examples we have of them are lamp-black, and (somewhat less perfect) charcoal, and gas-coke.

30. Newton's speculations on this subject, taken from the "Queries" at the end of his *Optics*, give an exceedingly interesting sketch of the state of human knowledge in his time. We quote a few of the more curious. There is a strange admixture of errors, but a still more strange anticipation of some of the most important of modern discoveries.

"Query 6. Do not Black bodies conceive heat more easily from light than those of other colours do, by reason that the light falling on them is not reflected outwards; but enters the bodies, and is often reflected and refracted within them, until it be stifled and lost?

"Query 8. Do not all Fixed bodies, when heated beyond a certain degree, emit light and shine; and is not this emission performed by the vibrating motions of their Parts. And do not all bodies, which abound with Terrestrial parts, and especially with Sulphureous ones, emit light, as often as those parts are sufficiently agitated; whether that agitation be made by heat, or by friction, or percussion, or putrefaction, or by any vital motion, or any other cause? . . .

"Query 9. Is not Fire a body heated so hot, as to emit light copiously? For what else is a red-hot iron than fire? And what else is a burning coal than red-hot wood?

"Query 10. Is not Flame a vapour, fume or exhalation heated red-hot, that is, so hot as to shine? For bodies do not flame without emitting a copious fume, and this fume burns in the flame. The *Ignis Fatuus* is a vapour shining without heat; and is there not the same difference between this vapour and flame, as between rotted wood shining without heat and burning coals of fire? In distilling hot spirits, if the head of the still be taken off, the vapour, which ascends out of the still, will take fire at the flame of a candle, and turn into flame, and the flame will run along the vapour from the candle to the still. Some bodies heated by motion or fermentation, if the heat grow intense, fume copiously; and if the heat be great enough, the fumes will shine, and become flame. Metals in fusion do not flame for want of a copious fume, except spelter, which fumes copiously, and thereby flames. All flaming bodies, as oil, tallow, wax, wood, fossil coals, pitch, sulphur, by flaming waste and vanish into burning smoke; which smoke, if the flame be put out, is very thick and visible, and sometimes smells strongly, but in the flame loses its smell by burning; and, according to the nature of the smoke, the flame is of several colours; as that of sulphur, blue; that of copper opened with sublimate, green; that of tallow, yellow; that of camphire, white. Smoke passing through flame cannot but grow red-hot; and red-hot smoke can have no other appearance than that of flame. . . .

"Query 11. Do not Great bodies conserve their heat the longest, their parts heating one another; and may not Great dense and Fixed bodies, when heated beyond a certain degree, emit light so copiously, as by the emission and re-action of its light, and the reflections and refractions of its rays within its pores, to grow still hotter, till it comes to a certain period of heat, such as is that of the sun? And are not the sun and fixed stars great earths vehemently hot; whose heat is conserved by the greatness of the bodies, and the mutual action and re-action between them, and the light which they emit; and whose parts are kept from fuming away, not only by their Fixity, but also by the vast weight and density of the atmospheres incumbent upon them, and very strongly compressing them, and condensing the vapours and exhalations which arise from them? . . . And the same great weight may condense those vapours and exhalations, as soon as they shall at any time begin to ascend from the sun, and make them presently fall back again into him; and by that action increase his heat, much after the manner that in our earth the air increases the heat of a culinary fire. And the same weight may hinder the globe of the sun from being diminished, unless by the emission of light, and a very small quantity of vapours and exhalations."

CHAPTER IV.

THEORIES OF PROPAGATION OF LIGHT.

31. We may begin by assuming that the sensation of light is due to a mechanical action on the retina.[1] Now such a mechanical action must have a mechanical cause, and, as far as we can judge with our present knowledge, the latter must consist of impacts on the retina, due to moving matter. This matter may have travelled all the way from the source of light, or it may have been set in motion in the eye by a disturbance (analogous to a wave) which has travelled from the source. What is transferred, or what moves, is a quite independent question. Light must, as far as we can conceive, consist in the motion of particles of some kind from external objects to the eye, or in the propagation of some disturbance or wave-motion in an as yet unknown medium. Though it has been proved, as we will presently show, that some of the consequences of the first supposition are entirely inconsistent with observed facts, the nature of the propagation of the supposed luminous particles is still a very interesting study, and indeed many of the fundamental

[1] [We make the assumption because the question involved belongs purely to Physiological Optics, a subject with which we do not profess to deal except to the very limited extent involved in the preliminary statements given in Chap. II. above.]

propositions in optics follow more easily from this hypothesis than from the other. We will therefore *not* at present dismiss this hypothesis, but will refer freely to it now and then, until its truth is shown to be inconsistent with experiment.

32. This hypothesis, which is associated with the names of Newton, Laplace, and Biot, is known as the *Corpuscular Theory* of light.

A very formidable objection to it, *in limine*, will be easily seen to be furnished by the speed of light. Since every point of every visible body must (on this theory) send such corpuscles to the eye, moving, as we shall find, at a rate of nearly 186,000 miles per second, their masses must be inconceivably minute in order that their united momentum may not amount in one second to something comparable with that of a cannon shot.

But, as we shall see, there are other grounds of objection, and such as no mere smallness of mass or size of each corpuscle can explain away. It must be allowed that not only does this theory give us the most simple explanation of some of the elementary facts (and, in consequence, even now enable us to investigate certain classes of phenomena with great simplicity as well as with accuracy), but that, in the hands of some of the extremely able men who maintained its truth, it was adapted with great skill to the explanation of many much more profound experimental results. It was obvious, however, that it had to be made more complex in its assumptions for each fresh fact that was forced under its sway, until it reached such a pitch of complexity that it must have of itself broken down, even if we had not been in possession of a rival theory.

33. The rival theory labours under considerable disadvantages, both from the scientific and from the popular point of view, inasmuch as the subject of wave-propagation is very much more obscure and difficult than that of the motion of free particles; but the student, who has mastered the fundamental difficulties of sound, which presents a fair although not an exact analogy, will find it comparatively easy to obtain a clear conception of the fundamental principles of the explanation offered by the *Undulatory Theory* of light. He will find that large classes of the more common phenomena can be explained on this hypothesis without any assumptions as to the nature of the wave-motion, other than the necessary characteristic of *periodicity;* and thus that, without any enforced complexity, half of the ground is already gone over. When, however, the phenomena of double refraction and polarisation have to be explained, the hitherto unlimited form of wave-motion has to be restricted, to vibrations in which no change of density of the vibrating medium can occur.

34. The difference between these two theories of light may be illustrated by contrasting wind moving at the rate of 1100 feet per second (80 corresponds to a violent hurricane), and sound, gentle or violent, moving at precisely the same rate—yet how different in its effects!

35. This brief mention of these rival and contrasted theories, with the extent to which each has succeeded in explaining phenomena, gives us a hint as to the future division of our subject. One large part is, to a great extent, equally well explained by either theory, being merely the mathematically-developed consequences of experimental facts which either theory is capable of explaining. Though some of the explanations given by

the corpuscular theory may be afterwards found, inconsistent with *other* facts of experiment, we need not for the time discard it. Our division will then be into two parts:—the first, based entirely on a few general experimental laws which follow from either theory, is merely a sort of extended geometry; the second, to which further experimental facts come in, enables us to select one of these theories as alone compatible with this further knowledge.

36. A simple illustration of the nature of this division will be found in the different conditions of fluid equilibrium according as we do not or do introduce the idea of action between the fluid and the containing vessel. In the first or hypothetical case it is known that the free surface must be horizontal, and that all its separate parts must lie in the same plane; in the second, *i.e.* the actual, case we find molecular action modifying these results, sometimes indeed to a very large extent, so that no part of the free surface is plane, and no two finite portions of it are the same level.

So in what is called GEOMETRICAL OPTICS it is assumed from experiment that light moves in straight lines in air, while PHYSICAL OPTICS, or the undulatory theory, agrees with experiment in showing that under certain circumstances a ray of light bends round an obstacle. But as, in obtaining the main facts of fluid equilibrium, capillary forces may be neglected, so, for the explanation of the ordinary phenomena of light, even with accuracy sufficient for the construction of the very finest telescopes and microscopes, it suffices that Geometrical Optics, based on laws *nearly* verified by experiment, be followed out to its consequences. The residual phenomena then come in to be treated by the undulatory theory, as some of them are found to be inconsistent with the corpuscular theory. Pouillet divides the subject, in consequence of this distinction, into two parts, viz. (1) that in which we deal with the direction only of the rays, and (2) that in which we deal with the physical properties of the rays themselves. This mode of division, however, is not a very accurate one, as will be seen when we deal with double refraction.

37. In this order we will consider the subject, giving the explanations of the approximate experimental laws of Geometrical Optics, as we reach them, in the language of either theory. But before we come to the residual phenomena we shall have found that the corpuscular theory must be rejected, and we will therefore give, with as much detail as is consistent with our limits, the principles of the undulatory explanation. The immediately succeeding chapters will therefore be devoted to Geometrical Optics.

CHAPTER V.

RECTILINEAR PROPAGATION OF LIGHT.

38. It is approximately true that, in any homogeneous medium, *Light moves in straight lines.* In this sense we speak of a *ray* of light. If the ray have a sensible cross-section, it is usual to speak of it as a *pencil of rays*, or simply a *pencil.*

If an opaque body be placed anywhere in the straight line between the eye and an object, the object is concealed. Through a long straight tube no objects can be seen but those situated in the direction of its axis produced. This is so fundamental a fact, or it is so evident a result of experience, that it is the foundation of every process which involves the direction in space of one object as regards another—whether it be for the aiming with a rifle, or for the delicate observations of a geodetic survey. But we must carefully observe the restrictions under which the statement is made. Not merely is it said to be only approximately true, but it is so only in a homogeneous medium. To both of these restrictions we will revert later.

39. On this is founded the geometrical theory of *Shadows*—a subject of some importance, especially as regards eclipses. In this application the results may be

considered as absolutely true, though, as we shall see in a subsequent page, the statement is liable in certain delicate cases to somewhat startling exceptions.

When an opaque body is placed between a screen and a luminous *point*, it casts a shadow on the screen. (The sun's image, formed by a lens or burning glass of short focus, is our best mode of attempting to realise the conception of a luminous point; but a fair approximation may be made by piercing a very small needle-hole in a large plate of thin metal, and placing it close to any bright flame or incandescent body.) The outline of the shadow is, of course, to be found by drawing straight lines from the luminous point so as to touch the opaque body all round. These lines form a cone. The points of contact form a line on the opaque body, separating the illuminated from the non-illuminated portions of its surface. Similarly, when these lines are produced to meet the screen, their points of intersection with it form a line which separates the illuminated from the non-illuminated parts of the screen.

40. This line is called the boundary of the *geometrical shadow*. A common but beautiful instance of it is seen when a very small gas-jet is burning in a ground-glass shade, near the wall of a room. In this case the cone, above mentioned, is usually a right cone with its axis vertical. Thus the boundary of the geometric shadow is a portion of a circle on the roof, but a portion of an hyperbola on the vertical wall. If the roof be not horizontal we may obtain in this way any form of conic section. Interesting and useful hints for the solution of problems in *Projection* may be obtained by observing the shadows of bodies of various forms cast in this way by

rays which virtually diverge from one point: *e.g.* how to place a plane (uncrossed) quadrilateral of given form so that its geometric shadow may be a square; how to place an elliptic disk, with a small hole anywhere in it, so that the shadow may be circular with a bright spot at its centre, etc.

41. When there are more luminous points than one, we have only to draw, separately, the geometrical shadows due to each of the sources, and then superpose them. A new consideration now comes in. There will be, in general, portions, of all the separate geometrical shadows, which overlap one another in some particular regions of the screen. In such regions we still have full shadow; but around them there will be other regions, some illuminated by one of the sources alone, some by two, etc., until finally we come to the parts of the screen which are illuminated directly by all the sources. There will evidently be still a definite boundary of the parts wholly unilluminated, *i.e.* the true shadow or *Umbra*, and also a definite boundary of the parts wholly illuminated. The region between these boundaries—*i.e.* the partially illuminated portion—is called the *Penumbra.*

42. Fig. 1 shows these things very well. It represents the shadow of a circular disk cast by four equally luminous points arranged symmetrically opposite to it as the corners of a square,—the disk being large enough to admit of a free overlapping of the separate shadows. The amount of want of illumination in each portion of the penumbra is roughly indicated by the shading. The separate shadows are circular, if the disk is parallel to the screen.

If we now suppose the number of sources to increase indefinitely, so as finally to give the appearance of a luminous *surface* as the source of light, it is obvious that

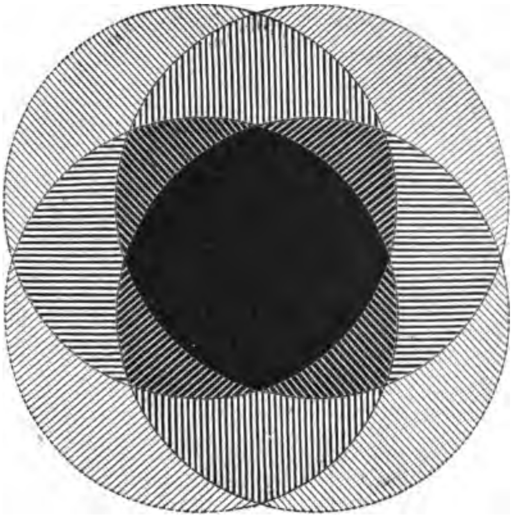

FIG. 1.

the number of degrees of darkness at different portions of the penumbra will also increase indefinitely, and the *abrupt* changes of illumination will merge into *steady* change; *i.e.* there will be a gradual increase of brightness in the penumbra, from total darkness at the edge next the geometrical shadow to full illumination at the outer edge.

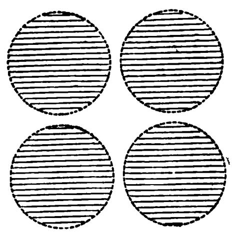

FIG. 2.

It is most instructive to contrast with the above figure that now given (fig. 2), in which the size of the disk is considerably diminished—everything else being unchanged. Here there is no true shadow—only four *equally bright* portions of the penumbra, each illuminated by three of the sources.

43. Thus we see at once why the shadows cast by the sun or moon are in general so much less sharp than those cast by the electric light (when it is not surrounded by a semi-opaque screen of porcelain or ground-glass). For, practically, at moderate distances from the electric arc, it appears as a mere luminous point. But, if we place a body at a distance of a foot or two only from the arc, the shadow cast will have as much of penumbra as if the sun had been the source. The breadth of the penumbra, when the source and screen are nearly equidistant from the opaque body, is equal to the diameter of the luminous source.

Simple as is the question from the point of view we have adopted, it may to some persons appear simpler to imagine themselves placed (as spectators) on the screen in different parts of the shadow or penumbra, and to consider what portions of the luminous source they would then be in a position to see.

44. This is the way in which we regard it when we observe an eclipse of the sun. When the eclipse is total, there is cast on the earth a real geometrical shadow—very small compared with the penumbra (for the *apparent* diameters of the sun and moon are nearly equal, but their distances are as 370 : 1); when the eclipse is annular, the shadow is all penumbra. In a lunar eclipse, on the other hand, the earth is the shadow-casting body, and the moon is the screen, and we observe things according to our first point of view.

45. Suppose, next, that the body which casts the shadow is a large one, such as a wall, with a hole in it. If we were to plug the hole, the whole screen would be in geometrical shadow. Hence the illumination of the screen by the light passing through the hole is precisely

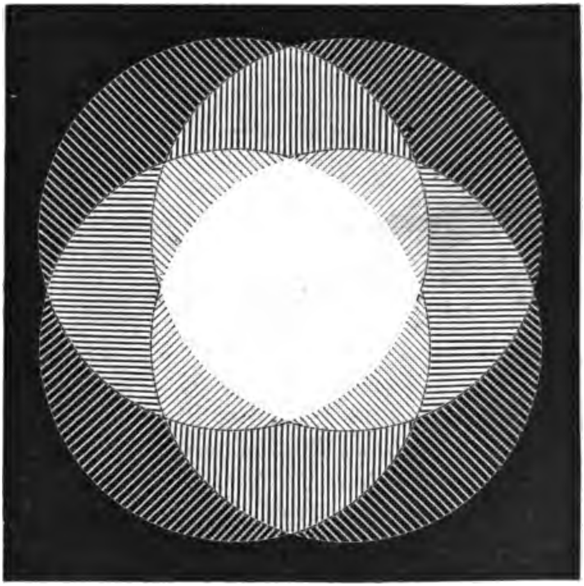

FIG. 3.

what would be cut off by a disk which fits the hole. Fig. 3, which is the complement of fig. 1, gives therefore the effect of four equal sources of light shining on a wall through a circular hole.

And it is evident that, with the change of a word here and there, the previous reasoning may be applied to this case also. The umbra in the former case becomes the fully illuminated portion, and *vice versâ*. The penumbra remains the penumbra, but it is now darkest where before it was brightest, and *vice versâ*.

For further information we subjoin the complement (fig. 4) of the second case above—the same four sources, but the smaller hole. Here we have four *equally* bright, separate images—one belonging to each of the sources.

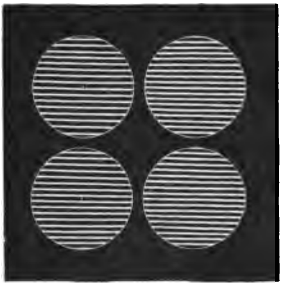

FIG. 4.

46. Thus we see how, when a *small* hole is cut in the window-shutter of a dark room, a picture of the sun, and of bright clouds about it, is formed on the opposite wall. This picture is obviously inverted, and also perverted, for not only are objects depicted lower the higher they are, but also objects seen to the right, when we look towards them, are found depicted to our right when we turn round and look at the front of the screen. But it will be seen unperverted (though still inverted) if it be received on a sheet of ground-glass and looked at from behind. The smaller the hole (so far, at least, as Geometrical Optics is concerned) the less confused will the picture be. As the hole is made larger the illuminated portions from different sources gradually overlap; and when the hole becomes a *window* we have no indications of such a picture except from a body (like the sun) much brighter than the other external objects. Here the picture has ceased to be one of the sun: it is now a picture of the window. But if the wall

could be placed 100 miles off, the picture would be one of the sun. To prevent this overlapping of images, and yet to admit a good deal of light, is one main object of the lens which usually forms part of the *Camera obscura.*

47. The formation of pictures of the sun in this way is well seen on a calm sunny day under trees, where the sunlight, penetrating through small chinks, forms elliptic spots on the ground. During a partial eclipse these pictures have, of course, a crescent form.

When detached clouds are drifting rapidly across the sun, we often see the shadows of the bars of the window on the walls or floor suddenly shifted by an inch or two, and for a moment very much more sharply defined. They are, in fact, shadows cast by a *small* portion of the sun's limb, from opposite sides alternately.

Another beautiful illustration is easily obtained by cutting with a sharp knife a *very small* T aperture in a piece of note-paper. Place this close to the eye, and an inch or so behind it place another piece of paper with a fine needle-hole in it. The light of the sky passing through the needle-hole forms a bright picture of the T on the retina. The eye perceives this picture, and in consequence receives the impression of the T much magnified, but *turned upside down.*

48. Another curious phenomenon may fitly be referred to in this connection, viz., the *Phantoms* which are seen when we look at two parallel sets of palisades or railings, one behind the other, or when we look through two parallel sides of a meat-safe formed of perforated zinc. The appearance presented is that of a magnified set of bars or apertures, which appear to move rapidly as we

slowly walk past. Their origin is the fact that, where the bars appear nearly to coincide, the apparent gaps bear the greatest ratio to the dark spaces; *i.e.* these parts of the field are the most highly illuminated.

The exact determination of the appearances in any given case is a mere problem of convergents to a continued fraction. But the fact that the apparent rapidity of motion of this phantom may exceed in any ratio that of the spectator, is of importance,—because it enables us to see how velocities, apparently of impossible magnitude, may be accounted for by the mere running along of *the condition of visibility*, among a group of objects no one of which is moving at an extravagant rate.

49. Another important consequence of the law of rectilinear propagation of light is that, *if the medium be transparent, the intensity of illumination which a luminous point can produce on a white surface directly exposed to it is inversely as the square of the distance.*

The word transparent implies that no light is absorbed or stopped. Whatever, therefore, leaves the source of light, must in succession pass through each of a series of spherical surfaces described round the source as centre. The same *amount* of light falls perpendicularly on all these surfaces in succession. The amount received in a given time by a unit of surface on each is therefore inversely as the number of such units in each. But the surfaces of spheres are as the squares of their radii,—whence the proposition. (We assume here that the speed of light is constant in the medium, and that the source gives out its light uniformly, and not by fits and starts; and we have also assumed, above, that light is unchanged in amount as it passes farther from the source.

Here we virtually assume the conservation of energy in one of its many forms.)

When the rays fall otherwise than perpendicularly on the surface, the illumination produced is proportional to the cosine of the obliquity; for the area seen under a very small given spherical angle increases as the secant of the obliquity, the distance remaining the same.

50. As a corollary to this we have the further proposition that *the apparent brightness of a luminous surface (seen through a transparent homogeneous medium) is the same at all distances.*

The word brightness is here taken as a measure of the amount of light falling on the pupil per unit of spherical angle subtended by the luminous surface. The spherical angle subtended by any small surface whose plane is at right angles to the line of sight is inversely as the square of the distance. So also is the whole light received from the surface. Hence the brightness is the same at all distances.

51. The word brightness is often used (even scientifically) in another sense from that just defined. Thus we speak of a bright star: we ask the question—When is Venus at its brightest? etc. Strictly, such expressions are not defensible except for sources of light which (like a star) have no apparent surface, so that we cannot tell from what amount of spherical angle their light appears to come. In that case the spherical angle is, for want of knowledge, assumed to be the same for all the sources, and therefore the brightness of each is now estimated in terms of the *whole* quantity of light we receive from it. Thus we speak of stars of the *first*, *second*, etc., *magnitude.*

This has obviously nothing to do with their size, it refers merely to the amount of light we receive from them.

52. It is in this sense only that we use the word when we speak of Venus at its brightest; for if we take the former definition of brightness, the solution of this once celebrated problem would be very different from that usually given. As the question, however, is an interesting one both in itself and historically, we give an approximate solution of it. The approximation assumes what is certainly not true, that the illuminated portion of Venus always appears *uniformly* bright, and of the same degree of brightness in all aspects.

Let a be the radius of the earth's orbit, b that of the orbit of Venus, both being supposed circular; δ the distance between the planets when Venus is brightest.

Then if θ be the apparent angular distance of the earth from the sun, as seen from Venus, the illuminated part of the disk of Venus as seen from the earth is only the fraction

$$\frac{1 + \cos\theta}{2}$$

of the whole disk. Hence the expression

$$\frac{1 + \cos\theta}{2\delta^2}$$

is to be made a maximum; subject to the obvious trigonometrical relation

$$a^2 = \delta^2 + b^2 - 2b\delta\cos\theta.$$

This can easily be effected by ordinary geometrical methods. But the shortest and simplest solution is by the differential calculus. Substituting in the expression for the brightness the value of $\cos\theta$, as given by this relation, and employing the usual process for finding a maximum, we obtain a quadratic equation to determine δ. Of this the only admissible root is the positive one

$$\delta = \sqrt{3a^2 + b^2} - 2b.$$

By means of this the other quantities can be calculated.

53. But another fact has to be taken into consideration when we apply the above definition of brightness in practice, at least if the object give a great deal of light. For the aperture of the pupil is usually very much contracted when we look at a brightly illuminated sky or cloud. Thus there is a rough compensation which, to a certain extent, modifies the effect on the retina.

54. Founded on the principles just explained is Cheseaux' celebrated argument about the finite dimensions of the visible universe. For it is easy to see, as below, that if stars be scattered through infinite space, with average closeness and brightness such as is presented by those nearest us, and if stellar space be absolutely transparent, the whole sky should appear of a brightness comparable to that of the sun. Cheseaux and Olbers, entertaining the idea that the universe of stars is infinite, endeavoured to show that, because the sky is not all over as bright as the sun, there is absorption of light in stellar space. This idea was ingeniously developed by Struve.

Consider a cone of small angle, whose vertex is at the spectator's eye. The volumes of parallel slices of such a cone, of equal thickness, are proportional to the squares of their distances from the vertex. The number of stars in each slice is proportional to the volume, and the whole light from them must be less as the square of the distance is greater. Hence every slice of the cone sends the same amount of light to the eye, provided that no star intercepts the light coming from another. This condition is unattainable, so that the conclusion is that the brightness is as great as it can be with the materials employed. Every portion of the background shines as if it were a star.

It is to be observed, however, that there are many considerations which, if taken into account, would seriously modify this conclusion. To mention only one—suppose there are, besides the incandescent stars, a very much larger number of dark ones (already cold, or not yet heated, it does not matter which). If these be distributed among the others, the calculation above would no longer be applicable.

55. A third very important fact, connected with our present subject, but not immediately deducible from our principle, is—*The brightness of a self-luminous or illuminated surface does not depend upon its inclination to the line of sight.*

Thus a red-hot ball of iron, free from scales of oxide, etc., appears flat, because uniformly bright, in the dark; so, also, the sun, seen through mist, appears as a flat disk. This fact, however, depends ultimately upon the second law of thermodynamics, and its explanation will be given later, when we deal with Radiation.

It may be stated, however, in another form, in which its connection with what precedes is more obvious—*The amount of radiation, in any direction, from a luminous surface is proportional to the cosine of the obliquity.*

56. The flow of light (if we may so call it) in straight lines from a luminous point, with constant speed, leads

as we have seen to the expression μ/r^2 (where r is the distance from the luminous point) for the quantity of light which passes through unit of surface (perpendicular to the ray) in unit of time, μ being a quantity indicating the rate at which light is emitted by the source. This represents the illumination of the surface on which it falls.

The flow through unit of surface whose normal is inclined at an angle θ to the ray is, of course,

$$\frac{\mu}{r^2}\cos\theta,$$

again representing the illumination.

These are precisely the expressions for the gravitation force exerted by a particle of mass μ on a unit of matter at distance r, and for its resolved part in a given direction. Hence we may employ an expression

$$V=\Sigma\frac{\mu}{r},$$

which is exactly analogous to the gravitation or electric potential, for the purpose of calculating the effect due to any number of *separate* sources of light. Hence every investigation connected with electric or gravitation attraction has an analogue in the theory of illumination. And thus, as we may often obtain the solution of a formidable question in illumination by thinking of the (known) solution of the corresponding problem in the other subjects, so the illumination problem often reduces to extreme simplicity some of the more formidable difficulties of the corresponding gravitational or electric problem.

57. Thus the fundamental proposition in potentials,

viz., that, if n be the external normal at any point of a closed surface, the integral

$$\iint \frac{dV}{dn} dS ,$$

taken over the whole surface, has the value

$$-4\pi\mu_0 ,$$

where μ_0 is the sum of the values of μ for each source lying *within* the surface, follows almost intuitively from the mere consideration of what it means as regards light. For from this point of view the integral represents the excess of the amount of light which has gone *into* the surface, over that which has gone *out*, in unit of time. Now it is clear that every source *external* to the closed surface sends in light which goes out again. But the light from an internal source goes wholly out; and the amount per second from each unit source is 4π, the total area of the unit sphere surrounding the source.

It is well to observe, however, that the analogy is not quite complete, so far as illumination is concerned. To make it so, all the sources must lie on the same side of the surface whose illumination we are dealing with. This is due to the fact that, in order that a surface may be illuminated at all, it must be capable of *scattering* light, *i.e.* it must be to some extent opaque. Hence the illumination depends mainly upon those sources which are on the same side as that from which it is regarded.

[Though this process bears some resemblance to the heat analogy employed by Sir W. Thomson for investigations in statical electricity (*Cambridge Mathematical*

Journal, 1842) and to Clerk-Maxwell's device of an incompressible fluid without mass (*Cam. Phil. Trans.*, 1856), it is by no means identical with them. Each method deals with a substance, real or imaginary, which flows in conical streams from a source so that the same amount of it passes per second through every section of the cone. But in the present process the speed is constant and the density variable, while in the others the density is virtually constant and the speed variable.]

58. We have said that light moves in straight lines in a homogeneous medium. This rectilinear path follows at once from the corpuscular theory, as well as from the undulatory theory of light: in the first case there is no deflecting cause, so each corpuscle moves in a straight line; in the second, the direction of propagation of a plane wave in an uniform isotropic medium is always perpendicular to its front. [It will be seen later that the word *isotropic* is essential to the general accuracy of this statement.]

59. Looking along a hot poker, or the boiler of a steamboat, we see objects beyond *distorted; i.e.* we no longer see each point in its true direction; and thus the light no longer moves in straight lines. Here we have a non-homogeneous medium, the air being irregularly expanded in the neighbourhood of the hot body. To this simple cause, want of homogeneity, are due the phenomena of mirage, the fata morgana, the duplication of images of a distant object seen through an irregularly heated atmosphere, the scintillation or twinkling of stars, and the uselessness of even the best telescopes at certain times, etc. It is interesting to note here, with reference to the recently established "mountain" observatories,

that Newton[1] says:—"Long telescopes may cause objects to appear brighter and larger than short ones can do; but they cannot be so formed as to take away that confusion of the rays, which arises from the tremors of the atmosphere. The only remedy is a most serene and quiet air, such as may perhaps be found on the tops of the highest mountains, above the grosser clouds."

60. The principle above explained suggests many simple methods of comparing the amounts of light given by different sources.

If, for instance, a porcelain plate, or even a sheet of paper, of uniform thickness, have one half illuminated directly by one source of light, the other by a different source, and if one or other of these sources be moved to or from the plate till the halves appear equally illuminated, it is obvious that the amounts of light given out by the two sources must be directly as the squares of their distances from the screen. This is the principle of Ritchie's photometer.

Rumford suggested the comparison of the intensity of the shadows of the same object thrown side by side on a screen by the two lights to be compared. In this case the shadow due to one source is lit up by the other alone; and here again the amounts of light given out by the sources are as the squares of their distances from the screen when the shadows are equally intense. The shadow-casting object should be near the screen, so as to avoid penumbra as much as possible; yet not too near, so that the two shadows may not overlap.

61. Bunsen has recently suggested the very simple expedient of utilising a grease-spot on white paper for

[1] *Optics*, Book I., end of part i.

photometric purposes. When the paper is equally illuminated from both sides, the grease-spot cannot be seen except by very close inspection. In using this photometer the sources are placed in one line with the grease-spot, which lies between them and can be moved towards one or other. To make the most accurate determinations with this arrangement, the adjustment should first be made from the side on which one source lies, then the screen turned round and the adjustment made from the side of the other source,—in both cases, therefore, from the same side of the paper screen. Take the mean of these positions (which are usually very close together), and the amounts of light emitted by the sources are as the squares of their distances from this point.

Wheatstone suggested a hollow glass bead, silvered internally, and made to describe very rapidly a closed path, for use as a photometer. When it is placed between two sources, we see two *parallel* curves of reflected light, one due to each source. Make these, by trial, equally bright; and the amounts of light from the sources are, again, as the squares of the distances.

62. These simple forms of apparatus give results which are fairly accurate, so long at least as the *qualities* of the light furnished by the two sources are nearly the same. But, when we endeavour to compare differently coloured lights, the result is by no means so satisfactory. In fact, we cannot well define equality of illumination when the lights are of different qualities.

In the undulatory theory, no doubt, we can distinctly define the intensity of any form of radiation. But the definition is a purely dynamical one, and has not necessarily any connection with what we usually mean by intensity, viz. the amount of effect produced upon the nerves of the retina, which is a question of physiological optics. Thus the theoretical intensities of a given violet and a given red source may be equal, while one may appear to the eye very much brighter than the other. Think, for instance, of a colour-blind person, who might, under conceivable circumstances, be unable to see the red at all. We are *all*, as it were, colour-blind as far as regards radiations whose wave-lengths are longer or shorter than those included in the range of the ordinary solar spectrum.

63. Other modes of measuring the intensity of light usually depend upon more recondite physical principles,—such as, for instance, the amounts of chemical action of certain kinds which can be produced by an exposure of a given duration to the light from a particular source. But all have the same grand defect as the simpler processes—they are not adapted to the comparison of sources giving different qualities of light. And those last mentioned are liable to another source of error, viz. the action of radiations which are not called light, only because they are not visible to the eye; for in all other respects they closely resemble light, and are often more active than it is in producing chemical changes.

CHAPTER VI.

SPEED OF LIGHT.

64. Light moves in interplanetary spaces with a speed of nearly 186,000 miles per second.

Of this we have four perfectly distinct kinds of proof, each of which depends on a method which is capable of giving pretty accurate results.

65. *Römer's Method.*—By this the finite speed of light was discovered in 1676.

Suppose, to illustrate this, that at a certain place a cannon is fired, precisely at intervals of an hour, and that the weather is perfectly calm. A person provided with an accurate watch travels about in the surrounding district. When he first hears the cannon let him note the time by his watch, then on account of the non-instantaneous propagation of sound, if at the next discharge he be *nearer* the gun than before, the report will arrive at his ear *before* the hour's interval has elapsed; if he be farther from the gun, the interval between the discharges will appear longer than an hour; and the number of seconds of defect or excess will evidently represent the time employed by sound in travelling over a space equal to the difference of his distances from the gun at the successive observations.

Now the satellites of Jupiter are subject—like our moon, only much more frequently—to eclipse. They move much faster, especially the nearer ones, than does the moon, so that their eclipses appear from the earth to take place almost *suddenly*, and the interval between two successive eclipses can easily be observed. The almost sudden extinction of the light is a periodic phenomenon similar to the discharge of the gun above imagined; and, if light take time to move from one place to another, we should find the interval between successive eclipses too short when we are approaching Jupiter, too long when we are receding from him. Such was found to be the case by Römer; and he also found that the shortening or lengthening of the interval depended upon the rate at which the earth was approaching to or receding from Jupiter. The inevitable conclusion from these facts is that light is propagated with finite speed. Römer calculated from them that light takes about $16^{m}.5$ to cross the earth's orbit. The exact speed deduced by this method is, after making all corrections, and assuming the most probable value of the solar parallax (which is our datum for measuring the dimensions of the earth's orbit), about 186,500 miles per second.

66. *Bradley's Method.*—This depends on the *Aberration of light*, discovered by Bradley in 1728.

When in a calm rainy day one stands still, he holds his umbrella vertically in order to protect himself. If he walk he requires to hold it forwards, and more inclined the faster he walks. In other words, to a person walking the rain does not appear to come in the same direction as it does to a person standing still.[1] Now the earth's speed in its orbit is a very large quantity, some $18\frac{1}{2}$ miles per second, or about $\frac{1}{10000}$th of that of light. Hence the light of a star does not appear to come in the proper direction unless the earth be moving exactly to or from the star, and, as the direction of the earth's motion is continually changing, so the directions in which different stars are seen are always changing, and thus this phenomenon, called the "aberration of light," proves at once the earth's motion round the sun and the finite speed of light.

As an additional illustration of the phenomenon, suppose a bullet to be fired through a railway carriage, in a direction perpendicular to the sides of the carriage. If the carriage be standing still, the bullet will make holes in the sides, the line joining which is perpendicular to the length of the carriage; if it be in motion, then the farther side of the carriage will have moved through a certain space during the interval occupied by the bullet in passing from side to side, and thus the line joining the holes in the sides (*i.e.* the line pursued by the bullet relatively to the carriage), will be inclined at an angle greater than a right angle to the direction of the train's motion.

It is evident that the path apparently described by each star during a year, in consequence of aberration, will be found by laying off from the star lines which bear the same ratio to the star's distance as the speed of the earth does to that of light,—their directions being always the same as that of the earth's motion at every

[1] In fact, to estimate the *relative* velocity of two moving bodies, treated as mere points, we must subtract the vector velocity of the first from that of the second.

instant. This is precisely the definition of the Hodograph of the earth's orbit. Hence, on account of the finite speed of light, each star appears to describe in space a circle (not an ellipse) of fixed magnitude, in a plane parallel to that of the ecliptic. As seen from the earth, therefore, stars will appear to describe paths which are the projections of these circles on the celestial sphere. These are in general ellipses, but they are circles for stars at the poles of the ecliptic and straight lines for stars on the ecliptic. These results are found to be quite consistent with observation; and the major axes of these ellipses, the diameters of the circles, and the lengths of the lines, all subtend equally angles of about 41″ at the earth. Hence the speed of light is to the speed of the earth in its orbit as $1 : \tan \frac{1}{2} 41''$; that is, 10,000 : 1.

It may, however, be said that aberration must depend on the earth's motion relative to the *star*, not on its motion relative to the sun. True; but the sun's motion is (we have every ground for supposing) sensibly the same both in direction and magnitude at least for periods of a few hundred years or so. The effect of this, whatever be its amount, is merely to effect a *constant* displacement of the star (which we have as yet only the very roughest modes of estimating) on which the aberration effect we have just discussed is simply superposed. [There are theoretical considerations of a different and much higher order of difficulty, connected with the relative motion of the earth and the luminiferous medium, which are raised by the problem of aberration. But they are not of a character suited to an elementary work like this.]

67. Both of the methods just given depend, for their final result, upon an exact knowledge of the earth's distance from the sun. But the most accurate measurements of this quantity are probably to be obtained from the speed of light itself, this being independently determined by the physical processes next to be described. Thus the earth's distance from the sun will probably in future be measured rather by the constant of aberration, or by the acceleration or retardation of the eclipses of Jupiter's satellites, than by a transit of Venus, by the moon's motion, or by the parallax of Mars. Thus Römer's and Bradley's processes, originally devised to find the speed of light, will be applied to the determination of solar parallax.

68. *Fizeau's Direct Measurement of the speed of Light.* —To illustrate the next and by far the most convincing *popular* proof of the finite speed of light, suppose a person looking at himself in a mirror, before which is moving a screen with a number of apertures, the breadth of each aperture being equal to the distance between each two of them. If the screen be at rest, with an aperture before the mirror, the light from the observer's face passes through the aperture and is reflected back, so that he sees himself as if the screen were not present. Suppose the screen to be moving in such a way that, when the light which passed through the aperture returns to the screen after reflection, the unpierced part of the screen is in its way, it is evident that the observer cannot see himself in the mirror. If the screen pass twice as fast, the light which escaped by one aperture will, after reflection, return by the next, so that he will see his image as at first. If three times as fast, the

second unperforated part of the screen will stop the returning light; so he cannot see his image.

To apply this practically, a thin metallic disk had a set of teeth cut on its circumference, so that the breadth of a tooth was equal to that of the space between two teeth. This disk could be set in very rapid rotation by a train of wheelwork, and the rate of turning could easily be determined by Savart's method; depending on the pitch of the note given by a spring pressing lightly on the teeth of one of the wheels. Light passed between two teeth to a mirror situated at 10 miles' distance, which sent it back by the same course, so that when the wheel was at rest the reflected light could be seen. On turning the disk with accelerated speed the light was observed to become more and more feeble up to a certain speed, at which it was extinguished; turning faster it reappeared, growing brighter and brighter till the speed was doubled; then it fell off, till it vanished when the speed was trebled, and so on. It is evident from the first illustration above that the speed of light in air is to that of the tooth, at the first disappearance of the reflected light, as the distance of the mirror from the disk is to the half breadth of the tooth.

It is not to be supposed that the description we have just given embodies all the details of this remarkable experiment. On the contrary, telescopes were used at each station to prevent loss of light as much as possible, and many other precautions were adopted which would be unintelligible to a reader unacquainted with the subjects to be treated farther on.

This method and its first results were published in 1849 in the *Comptes Rendus*. The experiments gave, on their very careful repetition by Cornu in 1874, the value 186,700 miles for the speed in vacuo.

69. *Foucault's Method.*—This was described in 1850 to the Academy of Sciences. It depends upon the principle of the rapidly-revolving plane mirror, introduced by Wheatstone to demonstrate the non-instantaneous propagation of an electric discharge. The mirror was made to rotate, about a diameter, from 600 to 800 times per second, by means of a *siren* driven by steam. A ray of sunlight fell upon it from a small aperture crossed by a grating of platinum wires. Between the wires and the mirror was placed an achromatic lens—the wires being farther from it than its principal focus, but not twice as far—so that the rays falling on the mirror were slowly convergent. After reflection they formed an image of the wires at a distance of about 4 metres from the mirror. In certain positions of the revolving mirror, the rays fell upon a concave mirror of 4 metres radius, whose centre of curvature was at the centre of the revolving mirror. They were, therefore, reflected back directly to the revolving mirror, and, passing again through the lens, formed an image of the wire grating which, when the adjustment was perfect, coincided with the grating itself. This coincidence was observed by reflection from a piece of unsilvered glass, placed obliquely in the track of the rays, the image in which was magnified by an eye-piece. It is obvious that, when the mirror is made to turn, the light which comes back to it after passing to the fixed mirror, finds it in a position slightly different from that in which it left it. That difference is due to the amount of rotation during the time of passage of the light to and fro along an air-space of 4 metres.

Accordingly, so soon as the mirror began to rotate with considerable speed, the coincidence between the wires and their images was destroyed; and the two were separated more and more widely as the speed of rotation was increased. It was easy to calculate, from the measured dimensions of the apparatus, the amount of deflection, and the rate of rotation of the mirror, the speed of light. The rate of rotation was calculated from the pitch of the note produced by the siren.

Foucault's early results with this apparatus showed that the speed of light which had been deduced from the old methods was too large; and he concludes his first paper by the statement that the determination of the distance of the earth from the sun must now be made by physical instead of astronomical methods. Foucault's process has recently been very considerably improved by Mitchelson, who, in 1879, found for the speed of light in vacuo 186,380 miles per second.

70. By interposing a tube filled with water, and having flat glass ends, between the fixed and revolving mirrors, Foucault found that (for the same rate of rotation) the displacement of the image was greater than before in the proportion of the refractive index of water to unity. Thus it was at once evident, by a mode of experimenting exposed to no possible doubt, that light moves faster in air than in water, and, therefore, as will be seen later, that the corpuscular theory of light must be abandoned.

71. Other methods of determining the speed of light in air, and for comparing the speeds of light in air and water (on which depends the most definite proof of the erroneousness of the corpuscular theory), and in still and

moving water, will be afterwards explained. They give results of very great value, but we cannot introduce them here, as they depend upon somewhat more recondite principles belonging to *physical* optics. They will be explained later.

72. It is interesting to observe that, as the nearest fixed star is probably about 200,000 times farther from us than the sun is, we now see such a star by light which left it more than three years ago. If, as is now supposed, *variable* stars (such as Mira Ceti) owe their rapid periodical changes of brightness to eclipses, and if different homogeneous rays travel with different speeds in free space, it is evident that such stars should show a gradual change of colour as they wax, and an opposite change as they wane. Nothing of the kind has as yet been observed, though it has been carefully sought for. Hence we have every reason to conclude that, in free space, all kinds of light have the same speed.

It will be seen later that *Dispersion* has been accounted for by the different speeds of light of different wave-lengths in the same refracting medium,—this being a consequence of the ultimate grained structure of ordinary matter, which is on a scale not incomparably smaller than the average wave-length.

CHAPTER VII.

BEHAVIOUR OF LIGHT AT THE COMMON SURFACE OF TWO HOMOGENEOUS MEDIA.

73. WHEN a ray of light, moving in one homogeneous medium, falls on the bounding surface of another homogeneous medium, it is in general divided into several parts, which pursue different courses. These parts are respectively:—(α) reflected; (β) refracted (singly or doubly); (γ) scattered; (δ) absorbed.

74. (α and β) In the first two categories the result is two (or three) rays of light pursuing definite paths according to laws presently to be given. The fraction of the incident light which is reflected is in general greater as the angle of incidence is greater: *i.e.* as the ray is *less* inclined to the reflecting surface. In one important class of cases the reflection is *total*. This will be discussed when we are dealing with refraction from one medium into another of smaller refractive index. But at direct incidence the reflected portion is much greater for some bodies, such as mercury, than for others, such as water or glass. In bodies which neither absorb nor scatter, the refracted portion of a ray consists of all the non-reflected portion, and therefore diminishes as the angle of incidence increases.

75. (γ) In the third category the common surface of the two media becomes illuminated, and behaves as if it were itself a source of light, sending rays in all directions. It may be objected to this, that in many cases the rays are scattered while penetrating the second medium. But in such cases the second medium cannot be called *homogeneous*. This case will come up for discussion when we treat of absorption and colours.

76. (δ) In the fourth category the light ceases (for an instant at least) to exist as light; but its energy may either become heat in the absorbing body, or it may again be given out by the absorbing body in the form of light, but of a *degraded* character. This is called *Fluorescence*, or *Phosphorescence*, according as the phenomenon is practically instantaneous or lasts for a measurable time.

77. In category (a) the light is sent back into the first medium; in (β) it penetrates into the second; in (γ) it goes, in general, mainly to the first; in (δ) it is shared by both.

78. It is by scattered light that non-luminous objects are, in general, made visible. Contrast, for instance, the effects when a ray of sunlight in a dark room falls upon a plate of polished silver, and when it falls on a piece of chalk. Unless there be dust or scratches on the silver you cannot see it, because no light is given from it to surrounding bodies except in one definite direction, into which (practically) the whole ray of sunlight is diverted. But the chalk sends light to *all* surrounding bodies from which any part of its illuminated side can be seen; and there is no special direction in which it sends a much more powerful ray than in others.

It is probable that if we could with sufficient closeness examine the surface of the chalk, we should find its behaviour to be of the nature of reflection, but reflection due to little mirrors inclined in all conceivable aspects, and at all conceivable angles, to the incident light. Thus scattering may be looked upon as ultimately due to reflection.

When the sea is perfectly calm, we see in it only one intolerably bright image of the sun. But when it is continuously covered with slight ripples, the definite image is broken up, and we have a large surface of the water shining by what is virtually scattered light—though it is really made up of parts each of which is as truly reflected as it was when the surface was flat.

79. We have spoken above of the behaviour of light at the common surface of two media. Now we do not by this phrase necessarily mean two media different in their chemical composition. We mean merely media *optically* different. Thus water with steam above it, and (in very special cases) layers of water or air of different temperatures, give surfaces of separation at which reflection and refraction may and do take place. But, except in such special cases, we rarely have an abrupt change, such as is necessary for reflection, between two portions of the same substance *in the same molecular state.* In general the transition is gradual; and special mathematical methods must be applied for the purpose of tracing the behaviour of the ray, which is now really travelling in one and the same *non-homogeneous* medium.

80. Having given this brief analysis of the phenomena at a common bounding surface of two homogeneous

media, we must now examine (in such detail as our plan admits) the laws according to which we determine the subsequent behaviour of the light belonging to each of the four categories above mentioned. We will also give, in each case, the more important consequences which follow from the experimental laws.

But there are two simple though very important consequences of the laws of reflection and ordinary refraction, which must be mentioned here.

81. First, the principle of *Reversal* of a ray. This merely asserts that if a ray, which has suffered any number of reflections and ordinary refractions, fall perpendicularly on a plane mirror, it will *retrace* exactly its original path. And it must not be confounded with the principle of *Reversibility* which we meet with in connection with thermodynamics and energy generally. For this latter principle would imply that the ray not only retraces its path, but returns to the source without either change of character or diminution of intensity—a result quite unattainable with any reflecting or refracting bodies yet met with.

The principle of reversal is sometimes very useful in our elementary investigations, especially about the action of prisms, etc.; but we must be cautious in using it for the higher parts of the subject.

On the corpuscular theory, this proposition is a fundamental one in dynamics of a particle.

On the wave theory, it is easily seen to follow from the fact that waves are propagated according to the same laws in any one direction, and in the opposite.

82. But another still more important proposition is suggested (and proved) by the mere consideration of

successive positions of the same wave, in all cases in which the wave-front is perpendicular to the ray, viz.—

If a set of rays have one wave-front, they will continue to have a common wave-front after any number of reflections or simple refractions; and the distance measured along the various rays, from any one wave-front to another, is the same at all parts of any one medium, through which the rays pass.

From the corpuscular point of view, this may be stated as follows:—If corpuscles of the same kind leave any surface in the direction of the normal, their paths, after any number of reflections or simple refractions, can be cut at right angles by a series of *parallel* surfaces.

In this form, the proposition is easily proved by a geometrical process. And a simple geometrical method, directly based on it, enables us easily to obtain the fundamental formulæ for mirrors, lenses, etc., as they will presently be given. We may leave this, however, to the reader's ingenuity—deriving our results, in the text, directly from the laws of reflection and refraction.

More generally, if l be the length of the part of a ray which lies within a medium whose refractive index is μ, the sum

$$\Sigma \mu l$$

is the same for each ray of the group between any two wave-surfaces. We shall find that this is an immediate consequence of the fact that the speed, in any medium, is inversely as the refractive index; so that the sum above is merely the *time* of passage of the light-disturbance from the one wave-surface to the other, and must therefore be the same along each ray. From the corpuscular point of view, μ is *directly* as the speed, so that the sum above expresses the Action (§ 181) of a corpuscle.

CHAPTER VIII.

REFLECTION OF LIGHT.

83. If light be reflected from a plane surface bounding two homogeneous isotropic media, *the incident and reflected rays are in one plane with, and are equally inclined (on opposite sides) to, the perpendicular to the reflecting surface at the point of incidence.*

This is sometimes stated in the form—*The angles of incidence and of reflection are equal to one another, and in one plane*:—these being *defined* as the angles made by the incident and reflected ray with the perpendicular (or, as it is often called, the *Normal*) to the surface.

The best experimental proof of the truth of this statement is deduced from the use of a reflecting surface of mercury in observations with the mural circle. The graduation of such an instrument is the most perfect that human skill can accomplish, and no one has ever been able to find by it the slightest exception to the preceding statement.

84. The principle of Hadley's quadrant, and of the sextant as now used (an invention of Newton's), is founded on this fact. If a plane mirror on which a ray falls be turned through any angle about an axis perpendicular to the plane of reflection, the reflected ray is

turned through twice that angle. This is an immediate consequence of the above law. For, if the plane be turned through any angle θ, the perpendicular to it is turned through the same angle. Hence the angle between the incident ray and the perpendicular is increased or diminished by θ, and therefore that between the incident and reflected rays (which is double of this) is altered by 2θ.

A plane mirror is now extensively used for the purpose of indicating, by the change of direction of a reflected ray, the motion of a portion of an instrument to which the mirror is attached. Thus the magnetometers of Gauss, the tuning-forks of Lissajoux, and the electrometers and galvanometers of Sir W. Thomson, are all furnished with mirrors. The law of reflection is also the basis of the goniometer, for the measurement of the angles of crystals and prisms.

85. It follows from this law that, *if a ray pass from one point to another, after any number of reflections at fixed surfaces, the length of its whole path from one point to the other is the least possible*—subject to the condition that it shall *meet* each of the reflecting surfaces.

For the point (in a given plane), the sum of whose distances from two given points (on the same side of the plane) is the least possible, is that to which, if lines be drawn from the points, they are in one plane with the normal (or perpendicular) to the given plane and make equal angles with it. That is, if one be the direction of an incident ray, the other is the direction of the corresponding reflected ray. And, as the same is true of each separate reflection, it is true for the whole course of the ray, since for any one of the reflecting surfaces may be substituted its tangent plane at the point of incidence.

86. It is to be remarked that there are exceptions to this form of the statement. The true form is that the actual path of a ray, under the given conditions, is less in length than any other path (satisfying the conditions) which is nowhere finitely divergent from it.

This may be best seen by another method. Suppose a series of ellipses to be described, whose foci are the source of light and an assigned point which is to be reached by the reflected ray. Let this system turn about the line joining the foci; it generates a series of prolate spheroids such that the time of light's passing from one focus to a point in any one of the surfaces, and thence to the other focus, is the same whatever point be chosen on that particular surface. If we take any point *without* that surface, for it the corresponding time is obviously greater. Hence to find the path of that one of a system of rays diverging from a given point which, after reflection at a given surface, shall pass through a second given point, we have only to imagine spheroids constructed as before. Of these one at least will *touch* the given surface. All points on the surface *in the neighbourhood of the point of contact* (mark this limitation) will in general be outside the spheroid. Hence this point gives the shortest path. But then the spheroid and the reflecting surface have the same tangent plane, and therefore the parts of the ray are equally inclined to the perpendicular to the surface.

87. We now arrive at a very important practical part of our subject, the formation of an *Image* by reflection at a plane surface. We will take the opportunity of making

some general remarks on the subject of images, which will enable us to avoid further digressions (of the same kind) in future chapters.

We may assume here—what is indeed evident from the rectilinear propagation of light—that objects are rendered visible to the eye by rays *diverging* from them. Hence, if we have a set of reflected or refracted rays diverging from any point, or diverging as if they came from any point, they will convey to the eye the impression of the existence of a luminous source at that point.

The eye, in fact, can only tell us what effect is produced upon it, *i.e.* what sort of mechanical action it is subjected to. Its indications must therefore depend only upon what reaches it, and in no other sense whatever upon the source or the path of light. This point from which rays diverge, or appear to diverge, is called an *image*.

[The remark above gives us an excellent instance of the way in which we are liable to deception by trusting to the direct (or uncontrolled) evidence of our senses. Some of the most perfect illusions which have ever been contrived depend solely upon the obvious fact above stated:—viz. that the eye (or any other organ of sense) can inform us only as to what reaches and affects it; not, in any way whatever, of whence or how that which affects it managed to reach it.]

88. *The image of any point in a plane mirror is found by drawing from the point a perpendicular on the mirror and producing it till its length is doubled.*

The extremity of the line so drawn is the image of the point; or, in other words, rays proceeding from the point to the mirror diverge, after reflection, as if they came from the image so found. The image in this case is called *virtual*, to distinguish it from cases, subsequently to be mentioned, where it is *real*—the distinction being that the rays have actually passed through a real image, while they only appear to come from a virtual one.

To prove this it is only necessary to observe that, if A (fig. 5) be a luminous point, or a point from which

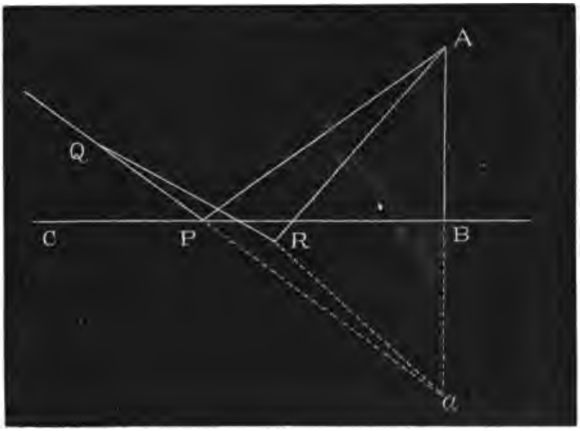

Fig. 5.

rays diverge, and CB any section of the mirror by a plane through AB, the perpendicular to it; and if we make B*a* = AB, and take any point P in BC, then, joining AP, *a*P, and producing the latter, the angles APB, *a*PB, and therefore CPQ, are equal; also the plane of the paper contains the perpendicular to the mirror at P. Hence if AP be any incident ray, PQ is the reflected ray, or the ray (which is *any one* from A, which reaches the mirror), after reflection, appears to come from *a*. Hence *a* is the image—a virtual one, as before noticed.

Also, if R be any point whatever (not P) in the plane of the mirror, we have obviously $a\text{R} = \text{RA}$. Hence the path AR, RQ is equal to aR, RQ, two sides of the triangle aRQ, of which aQ, which is equal to the actual path (AP, PQ), is the third side. Thus, as in § 83 above, the course of the reflected ray is the shortest possible.

89. Fig. 6 represents the pencils of diverging rays by

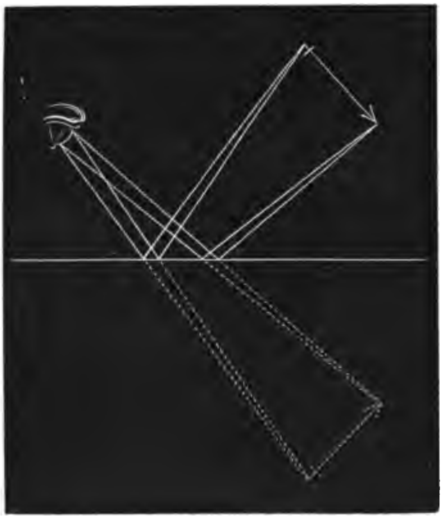

FIG. 6.

which two points of the image are rendered visible to an eye placed in front of the mirror. From the requisite modification of this figure it follows that one can see his whole person in a mirror of only half his height and breadth.

90. Dircks's *Ghost*, which has played a prominent part in popular entertainments for some years back, is the

image, in a large sheet of unsilvered plate glass hung at the front of the stage, of an actor or figure strongly illuminated, and concealed from the audience in a sort of enlarged prompter's box. Any one can see the phenomenon completely by looking into a plate-glass window on a sunny day, when he sees the passers-by apparently walking *inside* the house.

91. The principles already stated suffice fully for the explanation of the curious vistas of images formed by two *parallel* plane mirrors facing one another at opposite sides of a room. Each image formed by either mirror has its own image in the other; and so on indefinitely. These form a series whose successive distances from one another are alternately double the distance of the luminous point from one or other of the mirrors. The only additional observation necessary on this subject is that, if the mirrors are silvered on the back, the light at each reflection has to pass twice through the glass. Thus, if the glass be pinkish or greenish, the various images are more and more coloured as they are due to more numerous reflections.

92. These principles also easily explain the *Kaleidoscope* of Sir D. Brewster, where images are formed by two mirrors inclined to one another. It is easy to see that the series of images of a luminous point produced by such an arrangement after one, two, etc., reflections must all lie on a circle, whose plane passes through the point and is perpendicular to the line of intersection of the mirrors, in which line its centre lies; also that, if the angle between the mirrors be an aliquot part of four right angles, these images will form a finite number of groups, each consisting of an infinite number of images which have exactly the same position.

93. The explanation of the law of reflection which is furnished by the corpuscular theory is excessively simple. We have only to suppose that at the instant of its impact on the reflecting surface the velocity of a corpuscle *perpendicular* to the surface is reversed, while that *parallel* to the surface is unchanged. It bounds off, in fact, like a billiard-ball from the cushion. The undulatory theory gives an explanation, which is, in reality, quite as simple, but requires a little more detail for those who are not familiar with the common facts of wave-motion. We therefore reserve it for a time.

94. We must now consider the specially important case of reflection from a concave spherical surface. We call it specially important, because the reader who has once mastered the simple investigation which it requires will find very slight additional difficulty in passing to the case of a convex mirror, and even in following the somewhat more complex cases in which (at such surfaces) the ray is refracted instead of being reflected.

Let APB (fig. 7) be a section of a concave spherical

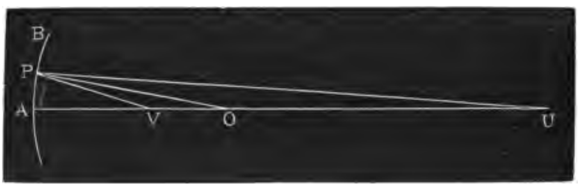

FIG. 7.

mirror by a plane passing through its centre of curvature O, and through the luminous point U. Then, if any ray from U, as UP, meet the surface, it will be reflected in a direction PV, such that UP, PO, and PV are in one plane, and so that PO bisects the angle UPV. (This follows because OP, a radius of the sphere, is normal to the surface at P.)

Hence it is *rigorously* true that, if V be the intersection of PV with UOA,

$$\frac{VO}{VP}=\frac{OU}{UP}.$$

The full consequences of this *exact* statement must be deferred for a little. For our present purpose an approximation will amply suffice.

95. Let us suppose P to be so near to A that no sensible error is introduced by writing A for P in the above formula. This amounts to supposing the mirror's breadth to be very small in comparison with its radius of curvature, while all the rays which we consider fall very nearly perpendicularly on the mirror. This is what we designate by the term *direct* (opposed to *oblique*) incidence. The formula now becomes

$$\frac{VO}{AV}=\frac{OU}{AU};$$

or, what is the same,

$$\frac{AO-AV}{AV}=\frac{AU-AO}{AU};$$

and V is, to the degree of approximation above stated, independent of the position of the point P.

96. If we call r the radius AO of the mirror, u = AU the distance of the source, and v = AV the distance of the point V, from the mirror, this becomes

$$\frac{r-v}{v}=\frac{u-r}{u}, \text{ or } \frac{1}{u}+\frac{1}{v}=\frac{2}{r}. \quad . \quad . \quad . \quad (a).$$

The formula, or the cut, shows at once that this relation between U and V is *reciprocal; i.e.* all rays from V, falling on the mirror, will be made to converge at U. These points are therefore called *Conjugate Foci.*

97. The simplicity of (a) is remarkable; so, also, is that of its interpretation. For the rays passing from a source to a given object, like the mirror, are less and less divergent as the source is farther off. Thus the reciprocal of the distance of a point from the mirror is the measure of the divergence (or convergence) of rays passing from it to (or coming to it from) the mirror. Hence (a) signifies that the (algebraic) sum of the divergences of the incident and reflected rays is equal to that divergence which the mirror can convert into parallelism.

In fact the rigorous geometrical relation may be written in the form $\angle AVP + \angle AUP = 2\angle AOP$,—which, when all three angles are small, is simply (a). And, in this way, we might have obtained the relation (a) at once. [A similar statement may easily be made in the case of refraction, § 142.]

98. Before we proceed to develop the consequences of this simple formula, we may point out that it is applicable to all cases of direct incidence:—to convergent rays falling on a concave mirror, to divergent rays falling on a convex mirror, etc. etc.

The reader may easily verify this for himself by trial. But it follows at once from the necessary interpretation of the negative sign in geometry. Thus, if the mirror (fig. 7) were convex, O would be to the *left* of A as the figure is drawn; and AO, if formerly positive, would now be negative. Thus, for a convex mirror, the formula is

$$\frac{1}{u}+\frac{1}{v}=-\frac{2}{r}.$$

If the incident rays be *convergent*, U is to the left of A, and therefore AU, or u, is negative; and so on.

99. We must now study the *relative* positions of U and V, in order to find the size and position of the image for different positions of the object.

Returning to the formula (a) above, we see that the following pairs of values of u and v satisfy it:—

u	v
Infinite.	$\frac{1}{2}r$.
Greater than r.	Greater than $\frac{1}{2}r$, less than r.
r	r
Less than r, greater than $\frac{1}{2}r$.	Greater than r.
$\frac{1}{2}r$.	Infinite.
Greater than 0, less than $\frac{1}{2}r$.	Negative.
0.	0.
Negative.	Greater than 0, less than $\frac{1}{2}r$.

100. Thus, when the source is at a practically infinite distance (as the sun or a star) the image is formed at a distance from the mirror equal to *half* its radius of curvature. It is then said to be in the *principal* focus. [Thus the focal length of the concave mirror of a reflecting telescope is half the radius of the sphere of which the mirror (approximately) forms a portion.]

As the source comes nearer, the image comes out to meet it, and they coincide at the centre of curvature of the mirror. [In fact, a ray leaving the centre of the mirror must meet the surface at right angles, and thus go back by the way it came.]

When the source comes still nearer, the image goes farther off, until, when the source is at the principal focus, the image is at an infinite distance; that is, the rays go off parallel to one another. This is the mode in which a concave reflector is employed for lighthouse purposes.

When the source comes still nearer, the image is behind the mirror, and therefore *virtual*:—*i.e.* the incident rays, are so divergent that part of their divergence remains after reflection.

This remnant of divergence becomes greater and greater as the source is nearer to the mirror, *i.e.* the (still virtual) image comes closer to the mirror, which finally behaves, for a very near source, almost precisely like a plane mirror.

A very simple method of obtaining the formula (a) is suggested by the wave-theory. Assimilate the spherical surface to a small portion of the vertex of an ellipsoid of revolution of which OA is the axis, O the centre of curvature at A, and U one focus. (See § 86.)

101. All of these phenomena can be beautifully seen in a dark room by employing a beam of sunlight, rendered distinctly visible, in the fashion described by Lucretius, by the motes in the air.

But they can also be instructively studied by holding a small bright object (such as a piece of chalk) in the axis of the mirror, and gradually varying its distance from the surface. By the necessary effort at accommodation with one eye (or by the stereoscopic effect with two), we obtain direct information as to the position, in space, of the image.

102. For further explanation, pictures are given (figs. 8, 9), showing the course of the pencil of rays when (1) a *real*, and (2) a *virtual*, image is formed by a concave mirror. It will be seen at once that, in the cases figured,

FIG. 8.

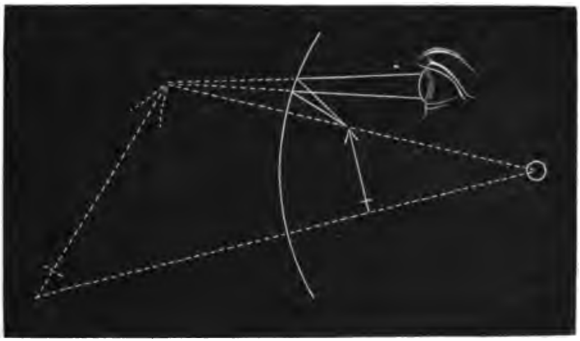

FIG. 9.

the *real* image is *inverted* and less than the object, the *virtual* image *erect* and larger.

In fact the size of a small object is obviously to that of its image in proportion to their distances from O, the centre of curvature of the mirror. Also the image is erect when it lies on the same side of the centre of curvature with the object; inverted, if on the opposite. In other words, the image is inverted if the rays cross one another's path, erect if they do not.

103. When the breadth of the mirror is large compared with its radius, the approximation upon which all these results depend can no longer be made. There is then no definite image even of a luminous point. It becomes spread over what is called a *Caustic*, a section of which is the bright curve familiar to every one who has looked at a cup of milk in sunshine.

104. As a very simple example of a caustic, which can be fully treated by elementary geometry, let us take

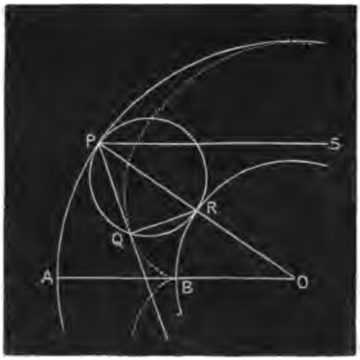

FIG. 10.

the case of a concave spherical mirror on which *parallel* rays fall (as from a very distant source).

Let AP, fig. 10, be a section of the mirror through its centre, O; the rays falling on it parallel to OA. Everything is, then, symmetrical about the line OA, so that if we find the section of the caustic surface by the plane of the paper, we have only to suppose the whole to rotate about OA to generate the surface.

Let SP be any one of the parallel rays in the plane of the paper, PQ the reflected ray. Join PO, and bisect it in R. On PR, as diameter, construct a circle; and describe another circle, with radius OR and centre O. Obviously these circles, which meet at R, *touch* one another at that point. Let the reflected ray, PQ, cut in Q the circle PQR. Since PO bisects the angle SPQ (by the law of reflection), and since SP is parallel to OA, the angle RPQ is equal to ROB. RPQ is subtended, at the *circumference* of a circle, by the arc QR; while ROB is subtended, at the *centre* of a circle of double radius, by the arc BR. Hence these arcs are equal; and if the circle PQR were to roll on the fixed circle RB, the point Q would ultimately coincide with the point B. But the point of contact, R, at any instant of the rolling, is for the moment fixed. Hence the motion of Q is, at every instant, perpendicular to RQ, *i.e.* it is *along the line* PQ. That is, PQ, the reflected ray, in every one of its positions *touches* the curve described by the point Q. This curve, indicated by the dotted line in the figure, and which is obviously an epicycloid, is therefore the section of the caustic surface by a plane passing through its axis. It has a cusp at the point B, which (as we have seen in § 100) is in this case the image of the infinitely-distant point, *i.e.* the principal focus of the mirror.

105. The same simple example gives us the means of explaining, by a particular case, a quite general property of any *small* pencil of rays, viz. that, when the pencil has not a focus, it has two *focal lines*, situated at different points in its course, and in directions at right angles to one another, and also to the axis of the pencil.

For it is obvious that every reflected ray, as PQ, passes accurately through the axis OA of the mirror. But Q is the intersection of two successive rays reflected in the plane of the figure; and, when the whole figure is made to rotate about OA, the path of Q is a circle which cuts the plane of the paper at right angles. Hence, if we confine our attention to rays which fall on a very small portion of the spherical mirror, inclosing the point P, every one of them after reflection will pass through a short line at Q perpendicular to the plane of the paper, and also through a short portion of the line OA; or, if we please, a short portion of a line in the plane POA perpendicular to PQ, where it meets OA.

106. This is a perfectly general proposition, easily established by the undulatory theory of light. For that theory shows (§ 82) that all the rays of any reflected, or simply refracted, pencil are normals to the wave-surface, as it is called. Now suppose that we take any small portion of a plane surface, with normals fixed to it at every point. These will, of course, form a set of parallel lines. But every curved surface, as geometry tells us, has two chief curvatures in sections through the normal and perpendicular to one another. Hence, to make our small plane surface fit the wave-surface of the pencil, we must bend it, in two planes perpendicular to one another, to the requisite amounts of curvature. The first bending makes all its normals (the rays of the corresponding pencil) pass through the axis of the cylinder into which it is bent. The second bending (without altering this state of matters except by lengthening or shortening the line intersected by all the normals) makes them all pass through the axis of a new cylinder, at right angles to the first.

107. Hence, to study fully the behaviour of a small pencil, we must take account of the position and length

of each of the focal lines; and then we have another problem, that of the *Circle of Least Confusion*, as it is called. This is the section of the pencil, between the two focal lines, in which the rays are most closely brought together, *i.e.* the section which will, in the absence of a true focus, most nearly satisfy the conditions of such a focus.

But questions of this kind require for their adequate treatment, except in special cases, much more of analysis than we can introduce here.

108. Even when the approximation of § 95 is close enough for ordinary purposes, it is not so for astronomical purposes, and the effect of its inexactness upon the image is known as *Spherical Aberration*. This is a mere special case of the general defect pointed out in § 103. For the fine mirrors of reflecting telescopes the spherical form cannot be employed; the surface of the mirror must be of *parabolic* section. For, as is easily seen from the fundamental property of the parabola, any ray falling on it parallel to the axis is reflected so as to pass exactly through the focus.

109. As a simple example of the application of the law of reflection at curved surfaces, when the rigorous solution is demanded, let us take the case of a vertical right cylinder, the object being a drawing on a horizontal plane. Such mirrors, with the frightfully distorted drawings necessary to give an image of natural proportions, were very common fifty years ago, but are now rarely seen. They are still, however, valuable as illustrations of our subject.

Let the plane of the object cut the axis OB of the cylinder at right angles in O (fig. 11), and let A be the position of the eye, and RQA a ray from a point R of the object, reflected at Q. Draw QP perpendicular to the axis. Then AQ and QR are in the same plane with QP (the normal to the surface) and make equal angles with it. Hence, when this figure is projected by vertical lines on the plane of the object, it takes the form in fig. 12, where AQ, QR now make equal angles with OQ. Also, if AB be drawn (in fig. 11) perpendicular to OP,

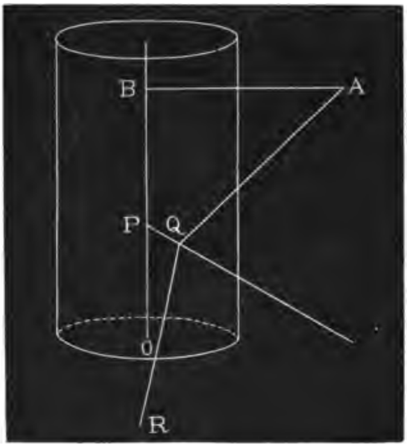

FIG. 11.

the ratio of AQ to QR in fig. 12 is equal to that of BP to PO in fig. 11.

Take $$QS : QO :: QR : QA,$$

and draw ST parallel to OA. Then it is obvious that

$$SR = ST = \frac{QS}{QO} OA;$$

and also that the angles QSR and QST are equal. Hence the following theorems, which enable us at once to draw a figure on the object plane such that its image shall appear of any assigned form.

1. Any line, such as QR, on the object plane, drawn from a point Q in the section of the cylinder so that the angles OQR and OQA are equal, is seen after reflection as a generating line of the cylinder.

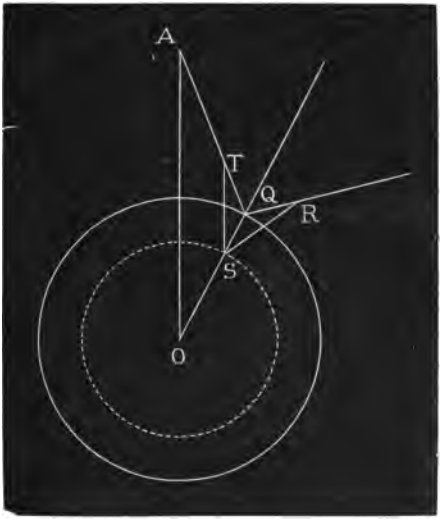

FIG. 12.

2. If an epicycloid be described by lines of fixed length OS, SR, turning about O and S, respectively, with angular velocities 1 and 2, and both at starting coinciding with OA in direction, its image will be a circular section of the cylinder.

Thus, if we imagine as drawn on the cylinder any number of vertical and horizontal sections, forming a network, the object corresponding to them can be traced as a number of intersecting straight lines and epicycloids. Thus we have a well-known means of drawing the required diagram; knowing, as we do, the small spaces on the diagram which correspond to given small spaces on the figure of the object to be seen by reflection. A similar process may be applied to other modes of using such mirrors.

110. When the cylinder has a small diameter, it may be usefully employed to intercept and reflect part of a beam of sunlight entering a dark room. It is easy to see, by a geometrical construction, that the reflected rays will, in this case, form a right cone, whose axis is that of the cylinder; and one of its generating lines will be parallel to the incident ray. Thus the angle of the cone becomes smaller as the inclination of the reflecting cylinder to the ray becomes less. If the ray, at the point of interruption, was at the centre of a spherical dome, after reflection it will form on the dome a circle, small or great, which passes through its original point of incidence.

Imitations, more or less perfect, of primary and secondary rainbows can easily be made by this process,—the sunbeam being led through a prism just before it falls on the cylindrical rod. This experiment is a very striking one; but, though capable of giving much information, it is of that dangerous kind which is liable to mislead instead of instructing an audience.

Another very beautiful illustration, simple in principle, but sometimes presenting considerable mathematical difficulties, is given by the continuous succession of images of the sun (or of some small, brilliant source) forming a luminous curve, produced by reflection at a polished wire of any form, which is made to move swiftly in a definite closed path.

111. If we look at a great number of thin cylindrical rods, parallel to one another, and illuminated by sunlight, the rays which reach the eye must, by what we have already said, each form a side of some right cone (of definite angle) whose axis is parallel to each of the cylinders. The appearance presented will therefore be that of a luminous *circle*, passing through the sun. Its angular diameter becomes less as the axes of the cylinders are less inclined to the incident rays.

This phenomenon is beautifully shown by some specimens of crystals, especially of Iceland spar, which are full of minute tubes parallel to one another. In a plate of such a doubly-refracting crystal, however, there are necessarily four images. That which is *throughout* due to the ordinary ray (this term will be explained later) shows perfectly the phenomenon above described. The light of the luminous circle is white. The other three curves are not circles, and in them the colours are separated. One of them, which is elliptical, is usually very much brighter than either of the remaining two.

CHAPTER IX.

REFRACTION OF LIGHT.

112. If homogeneous light be refracted at a plane surface separating two homogeneous isotropic media, *the incident and refracted rays are in one plane with the normal to the surface, they lie on opposite sides of it, and the sines of their inclinations to it are in a constant ratio to one another.*

The law of single refraction was put in a form equivalent to this (all but one word) for the first time by Snell in Leyden, some time before 1626. It was first published in 1637 by Descartes, who undoubtedly obtained it from Snell; but he gave it without any mention of its author.

113. The one word referred to is *homogeneous*, as applied to the incident light. For the fact that white light consists of innumerable different homogeneous constituents, which are separated from one another by refraction, was first established by Newton. We quote his own account of this extremely important discovery, from his letter to Oldenburg, dated Cambridge, Feb. , 167½:—

"To perform my late promise to you, I shall without further ceremony acquaint you, that in the year 1666 (at which time I applied myself to the grinding of optick-glasses of other figures than spherical) I procured me a triangular glass-prism, to try therewith the celebrated phænomena of colours. And in order thereto, having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the sun's light, I placed my prism at its entrance, that it might be thereby refracted to the opposite wall. It was at first a very pleasing divertisement, to view the vivid and intense colours produced thereby; but after a while applying myself to consider them more circumspectly, I became surprised to see them in an oblong form; which, according to the received laws of refractions, I expected should have been circular. They were terminated at the sides with streight lines, but at the ends the decay of light was so gradual, that it was difficult to determine justly what was their figure, yet they seemed semicircular.

"Comparing the length of this coloured *spectrum* with its breadth, I found it about five times greater; a disproportion so extravagant, that it excited me to a more than ordinary curiosity of examining from whence it might proceed. I could scarce think, that the various thickness of the glass, or the termination with shadow or darkness, could have any influence on light to produce such an effect: yet I thought it not amiss, first to examine those circumstances, and so tried what would happen by transmitting light through parts of the glass of divers thicknesses, or through holes in the window of divers bignesses, or by setting the prism without, so that the light might pass through it, and be refracted, before it was terminated by the hole; but I found none of those circumstances material. The fashion of the colours was in all these cases the same.

"Then I suspected, whether by any unevenness in the glass, or other contingent irregularity, these colours might be thus dilated. And to try this, I took another prism like the former, and so placed it, that the light passing through them both, might be refracted contrary ways, and so by the latter returned into that course from which the former had diverted it: for by this means I thought the regular effects of the first prism would be destroyed by the second prism, but the irregular ones more augmented, by the multiplicity of refractions. The event was, that the light, which by the first prism was diffused into an oblong form, was by the second reduced into an orbicular one, with as much regularity as when it did not at all pass through them. So that whatever was the cause of that length, it was not any contingent irregularity.

"I then proceeded to examine more critically, what might be effected by the difference of the incidence of rays coming from divers parts of the sun; and to that end, measured the several lines and angles belonging to the image. Its distance from the hole or prism was 22 foot; its utmost length 13¼ inches; its breadth 2⅝; the diameter of the hole ¼ of an inch. The angle which the rays, tending towards the middle of the image, made with those lines, in which they would have proceeded without refraction, was 44 deg. 56 min., and the vertical angle of the prism 63 deg. 12 min. Also the refractions on both sides the prism, that is, of the incident and emergent rays, were, as near as I could make them, equal; and consequently about 54 deg. 4 min. And the rays fell perpendicularly upon the wall. Now subducting the diameter of the hole from the length and breadth of the image, there remains 13 inches in the length, and 2⅜ the breadth, comprehended by those rays which passed through the center of the said hole; and consequently the angle of the hole, which that breadth subtended, was about 31 min. answerable to the sun's diameter; but the angle which its length subtended, was more than 5 such diameters, namely, 2 deg. 49 min.

"Having made these observations, I first computed from them the refractive power of that glass, and found it measured by the ratio of the sines 20 to 31; and then by that ratio I computed the refractions of two rays flowing from opposite parts of the sun's *discus*, so as to differ 31 min. in their obliquity of incidence, and found that the emergent rays should have comprehended an angle of about 31 min. as they did before they were incident.

"But because this computation was founded on the hypothesis of the proportionality of the sines of incidence and refraction, which though by my own experience I could not imagine to be so erroneous, as to make that angle but 31 min. which in reality was 2 deg. 49 min., yet my curiosity caused me again to take my prism: and having placed it at my window, as before, I observed, that by turning it a little about its axis to and fro, so as to vary its obliquity to the light, more than an angle of 4 or 5 degrees, the colours were not thereby sensibly translated from their place on the wall; and consequently by that variation of incidence, the quantity of refraction was not sensibly varied. By this experiment, therefore, as well as by the former computation, it was evident that the difference of the incidence of rays, flowing from divers parts of the sun, could not make them after decussation diverge at a sensibly greater angle, than that at which they before converged; which being at most but about 31 or 32 min., there still remained some other cause to be found out, from whence it could be 2 deg. 49 min.

"Then I began to suspect, whether the rays, after their trajection through the prism, did not move in curve lines, and according to their more or less curvity, tend to divers parts of the wall. And it increased my suspicion, when I remembered that I had often seen a tennis-ball, struck with an oblique racket, describe such a curve line. For, a circular as well as a progressive motion being communicated to it by that stroke, its parts, on that side where the motions conspire, must press and beat the contiguous air more violently than on the other, and there excite a reluctancy and re-action of the air proportionably greater. And for the same reason, if the rays of light should possibly be globular bodies, and by their oblique passage out of one medium into another acquire a circulating motion, they ought to feel the greater resistance from the ambient æther, on that side where the motions conspire, and thence be continually bowed to the other. But notwithstanding this plausible ground of suspicion, when I came to examine it, I could observe no such curvity in them. And besides (which was enough for my purpose) I observed, that the difference betwixt the length of the image and the diameter of the hole, through which the light was transmitted, was proportionable to their distance.

"The gradual removal of these suspicions at length led me to the *experimentum crucis*, which was this. I took two boards, and placed one of them close *behind the prism at the window*, so that the light might pass through a small hole, made in it for the purpose, and fall on the other board, which I placed at about 12 feet distance, having first made a small hole in it also for some of that incident light to pass through. Then I placed another prism behind this second board, so that the light trajected through both the boards might pass through that also, and be again refracted before it arrived at the wall. This done, I took the first prism in my hand, and turned it to and fro slowly about its axis, so much as to make the several parts of the image, cast on the second board, successively pass through the hole in it, that I might observe to what places on the wall the second prism would refract them. And I saw, by the variation of those places, that the light, tending to that end of the image towards which the refraction of the first prism was made, did in the second prism suffer a refraction considerably greater than the light tending to the other end. And so the true cause of the length of that image was detected to be no other, than that *light* is not similar or homogeneal, but consists of *difform rays, some of which are more refrangible than others;* so that without any difference in their incidence on the same medium, some shall be more *refracted* than others; and therefore that, according to their *particular degrees of refrangibility*, they were transmitted through the prism to divers parts of the opposite wall."

We have quoted this passage at length, because it shows, in a manner intelligible to all, the way in which a true experimenter goes to work on a novel difficulty; as well as the singular amount of new insight into other, and apparently unconnected questions, which a master in physics obtains from the consideration of any one problem.

114. The *constant ratio* mentioned in the above statement (§ 112) of the law of refraction is now called the *refractive index.* Its numerical value depends upon the nature of the two media, and also upon the quality of the homogeneous light. It is usually greater for orange light than for red, for yellow than for orange, and so on,—so that the violet rays are often called the "more refrangible" rays.

This statement is, however, liable to some very singular exceptions, which will be mentioned later, when we are dealing with anomalous dispersion.

115. The following experimental facts may be regarded, some as additions to, some as mere consequences of the law.

When refraction takes place from a rarer into a denser medium, the angle of refraction is usually less than that of incidence, *i.e.* the refractive index is greater than unity.

If the refractive index for a particular kind of light from a medium A into another B be μ, that from B to A is $1/\mu$. In other words, a refracted ray may be sent back by the path by which it came (§ 81).

If μ_1 be the refractive index for a particular ray from A into B, and μ_2 that for the same ray from A into C, that from B into C is μ_2/μ_1.

116. These being premised, let us consider a point-source of homogeneous light in air, shining on a surface of water. Here we may take μ as on the average about equal to $1 \cdot 33$ or, roughly, $\frac{4}{3}$.

Let MN (fig. 13) be the perpendicular to the water surface at the point where the incident ray AP meets it. In the plane APM make the angle QPN such that

$$\sin APM = \tfrac{4}{3} \sin QPN ,$$

then PQ is the refracted ray. If QP be produced back-

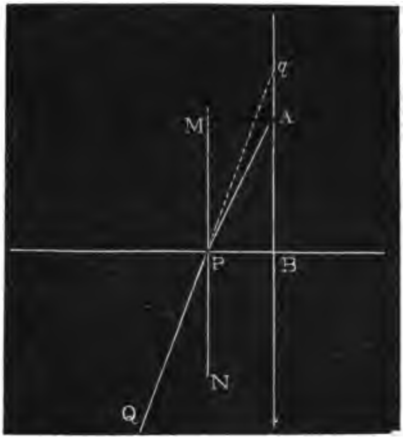

FIG. 13.

wards to meet the vertical line BA in q, we may present this statement in the form

$$\frac{PB}{PA} = \frac{4}{3}\frac{PB}{Pq}, \text{ or } PA = \tfrac{3}{4}Pq.$$

If the rays fall *nearly* perpendicularly on the surface, we may put (approximately, § 95) B for P, and we have

$$Bq = \tfrac{4}{3}BA.$$

Hence, an eye placed under water and nearly in the vertical through A, sees a *virtual image* of A at q, one-third farther from the surface of the water.

117. As P is taken farther and farther from A, the angle of incidence becomes more nearly a right angle, and the sine of the angle of refraction becomes more nearly equal to $\tfrac{3}{4}$. Hence we see that:—

A ray cannot go from air into water so as to make, with the perpendicular to the surface, an angle whose sine is greater than $\tfrac{3}{4}$.

The true nature of this most important statement is, however, best seen when we suppose the source to be

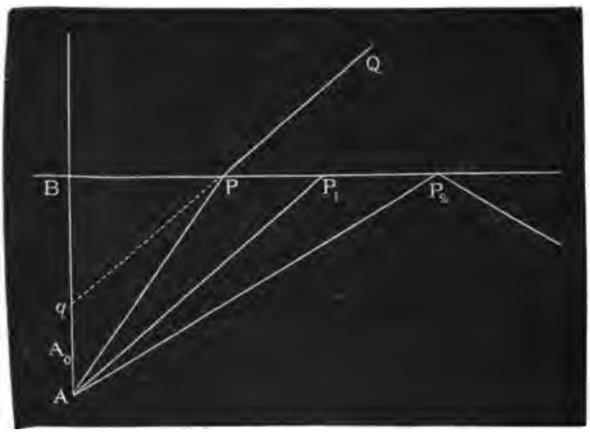

Fig. 14.

under water, and the light to be refracted into air. If APQ (fig. 14) be the course of a ray, we have as before, remembering that (§ 115) the refractive index has now the reciprocal of its former value,

$$AP = \tfrac{4}{3}Pq.$$

Hence if P_1 be taken so that

$$AP_1 = \tfrac{4}{3}P_1B,$$

it is clear that q coincides with B, or the ray AP_1, refracted at P_1, runs along the surface of the water.

If AP_2 be less than $\frac{4}{3}$ P_2B, no point q can be found; so that the ray AP_2 *cannot get out of the water.* It is found to be completely reflected in the water. This reflection, unaccompanied by refraction, is called *Total Reflection.*

The limiting angle of incidence (at P_1) which separates the totally reflected rays from those which (at least partially) escape into air is called the *Critical Angle.*

When an equilateral triangular prism of glass is placed in a ray of sunlight, and made to rotate, we see on a properly adjusted screen (besides the spectra formed by refraction) beams of white light reflected alternately from the *outside* and from the *inside* of each face. The totally reflected ray from the inside is seen at a glance to be very much brighter than that reflected from the outside.

118. To an eye placed nearly in the vertical of A, and outside the water, A appears at A_0, where

$$A_0B = \tfrac{3}{4}AB.$$

Thus a clear stream, when we look vertically into it, appears to be of only $\frac{3}{4}$ths of its real depth. But when we look more and more obliquely, seeing A for instance by the ray QP, the image appears to rise nearer and nearer to the surface; or, if A be at the bottom, the water will appear more and more shallow; and all objects in it will appear to be crowded towards the surface.

Thus if part of a stick be immersed in water, that part appears bent up towards the surface of the water.

119. To obtain a more accurate idea of what takes

place in this case, we must investigate the form of the caustic. This happens to be a very easy matter. Refer to fig. 15, where the points A, A_0, B, P, P_1, Q, q are the same as in fig. 14.

Produce AB, so that BA′ = AB; and, P being the point of incidence of a ray, draw the circle AA′P. Let the refracted ray PQ, produced backwards, meet this circle in R. Join RA, RA′. Then the angle at R is

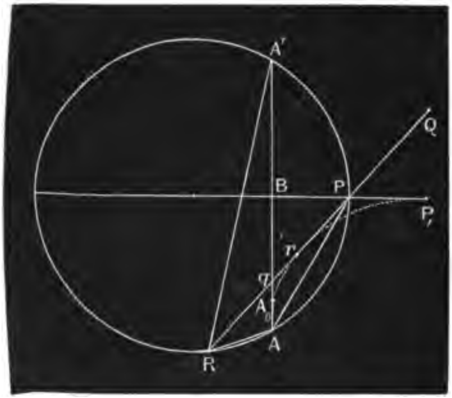

Fig. 15.

evidently bisected by RP, and each of its halves is equal to A′AP, the angle of incidence. Also the angles at q are the angle of refraction, and its supplement. Thus, in consequence of the law of refraction,

$$\frac{RA}{Aq} \text{ and } \frac{RA'}{A'q} \text{ are each} = \mu.$$

Thus $AR + RA' = \mu AA'$, and the locus of R is an ellipse whose foci are A and A′. RP is *normal* to the ellipse, because it bisects the angle between the focal distances of R. Thus the caustic is the *evolute* of this ellipse, and is represented by the dotted curve in the figure. The branch of it which we require touches BA at A_0, and BP at P_1.

The focal lines of the refracted pencil are therefore at q and r in the figure. And a glance at the figure shows why an object under water appears to be raised nearer to the surface, and also to come nearer to the

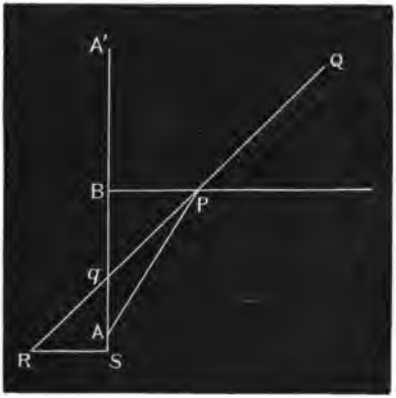

FIG. 16.

spectator, as his eye, Q, is brought nearer to the surface of the water.

120. Another mode of obtaining the same result, by the principle laid down in § 82, is of interest. Suppose the general wave, of which Q is a point, to have its motion reversed; and suppose the water done away with. Then in the time in which light originally described the path AP, PQ (fig. 16), it would (in air) go back along QP produced to a point R, such that

$$PR = \mu PA,$$

and the wave-surface at R will of course be perpendicular to RP. Draw RS perpendicular to BA, and we see by the law of refraction that

$$\mu^2 Bq = BS ,$$

whence the locus of R is the ellipse, as before.

121. Again, to an eye at A (fig. 14), all objects *above* the water will be seen within a right cone of which AB is the axis and AP_1 a side. The rest of the water surface, outside the cone just mentioned, shows us objects at the bottom by reflection in a perfect mirror.

122. The whole of what precedes is a mere application of the equation

$$\sin \alpha = \mu \sin \beta \quad . \; . \; . \quad (1)$$

between the angles of incidence and refraction. Some further consequences, which will be of use to us later, may now be deduced.

Take any line, AB (fig. 17), and divide it, internally at C, and externally at C′, in the ratio $1 : \mu$. Describe the circle whose diameter is CC′. Then geometry shows at once that, if P be any point in this circle,

$$AP : PB :: 1 : \mu .$$

Hence $\alpha = \angle OAP$, $\beta = \angle OBP$, satisfy (1) above; and are therefore corresponding values of the angles of incidence and refraction, when the refractive index is μ.

If AD be drawn perpendicular to AB, BD touches the circle, and ABD is the critical angle (§ 117).

123. Let PB, PA, cut the circle in p, q, respectively. Then, by geometry, the arcs Cp, Cq are equal. Hence COp, at the centre, is equal to qPp at the circumference, *i.e.* to $\alpha - \beta$; and it obviously increases as P, and there-

fore p, moves towards D. That is, *the change of direction produced by one refraction increases with the angle of incidence.*

124. Let Bp'P' be a proximate position of BpP. Then when P moves to P', α increases by the angle at the circumference on arcs PP' + pp', β by the angle on PP' − pp'. Hence $\alpha - \beta$ increases by the angle on $2pp'$. Now as P moves towards D, pp' and PP' tend constantly

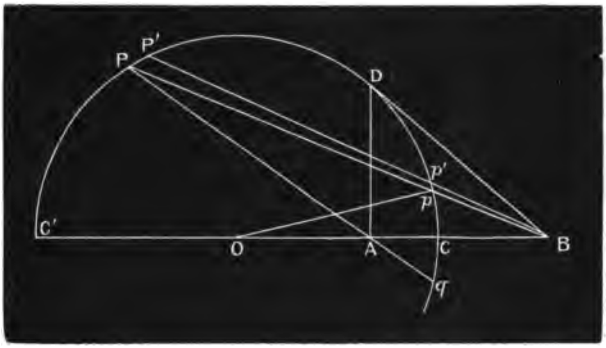

FIG. 17.

to equality. *Hence $\alpha - \beta$ increases faster and faster, whichever of α or β increases uniformly.* This result will be useful when we are dealing with prisms (§ 132).

125. Again the increase of $2\beta - \alpha$, *i.e.* $\beta - (\alpha - \beta)$, is in the same case the angle at the circumference on the arc

$$PP' - 3pp'.$$

While P is near to C', we have approximately

$$PP' : pp' :: BC' : BC :: \mu + 1 : \mu - 1.$$

Again, when P is near D, PP' and pp' tend to equality. Thus, so long as $1 > 3\dfrac{\mu - 1}{\mu + 1}$ (or $\mu < 2$), $2\beta - \alpha$ begins at zero at C′, increases till $PP' = 3pp'$, and thenceforward diminishes. Thus $2\beta - \alpha$ *has its maximum value* when $BP = 3Bp$, or (since $\angle PpO = \alpha$, and $OB = \mu \, . \, Op$)

$$2 \cos \alpha = \mu \cos \beta.$$

This, combined with (1) of § 122, gives at once, as the value of α for the maximum of $2\beta - \alpha$,

$$3 \cos^2\alpha = \mu^2 - 1.$$

We will refer to this when we are dealing with the primary rainbow (§ 157).

126. All that precedes has been obtained on the supposition that the light we are dealing with is homogeneous. But when white (*i.e.* compound) light is emitted by A (fig. 14), the image-point A_0 will be nearer the surface for each constituent the greater is the corresponding refractive index. Thus a white point at A will appear to be drawn out into a coloured line whose lower end is red and its upper end violet. Any one can verify this phenomenon by looking obliquely, in a sunny day, at a small white pebble or shell under water.

127. It is easily seen from the law of refraction that light, on passing through a plate of homogeneous material with parallel faces, finally emerges in a direction parallel to that at incidence, and that, therefore, white light comes out from it still white. If the plate be water in a vessel with thin parallel glass sides, a body placed close to one side, while the eye is close to the other, appears to be at $\frac{3}{4}$ of its real distance from the eye.

128. The complete explanation of the law of refraction, on the corpuscular theory, was given by Newton. It is still of importance, as the earliest instance of the solution of a problem involving what are called mole-

cular forces. Newton showed that, as the molecular forces on a corpuscle balance one another at every point inside either of the media, its speed must be constant in each; but that, in passing through the surface of separation of the two media, the square of the speed perpendicular to the surface undergoes a finite change.

Thus, if v be the speed in air, a the angle of incidence, then in denser media, such as water or glass, the speed parallel to the surface is still $v \sin a$, but that perpendicular to the surface is no longer $v \cos a$, but $\sqrt{v^2 \cos^2 a + a^2}$, where a is a quantity depending on the medium, and also on the particular species of light. Thus the whole speed is $\sqrt{v^2 + a^2}$; and if β be the angle of refraction,

$$\sqrt{v^2 + a^2} \sin \beta = v \sin a.$$

This is formally the statement of fact in § 112.

129. When the surfaces of a piece of glass, etc., are plane, but not parallel, we have what is called a *Prism*.

The general nature of the action of a prism will be easily understood by the help of the previous illustrations, if we restrict ourselves, for the moment, to the case of a prism of very small angle and to rays passing nearly perpendicular to each of its faces. Thus (§ 116), the rays falling nearly at right angles to its surface from a point A (fig. 18) will, after the first refraction, appear to diverge from a luminous line RV, red at the end next to A, violet at the other. This line is in the perpendicular AB from A to the first surface of the prism; and BR = μBA, if μ be the refractive index for red rays. Draw from R and V perpendiculars RS, VT to the second surface of the prism. Join BS, BT, and draw Ar, Av parallel to them so as to cut respectively RS in r and

VT in v. To an eye behind the prism, the bright point A will appear to be drawn out into a coloured line rv, red at the end nearest to A.

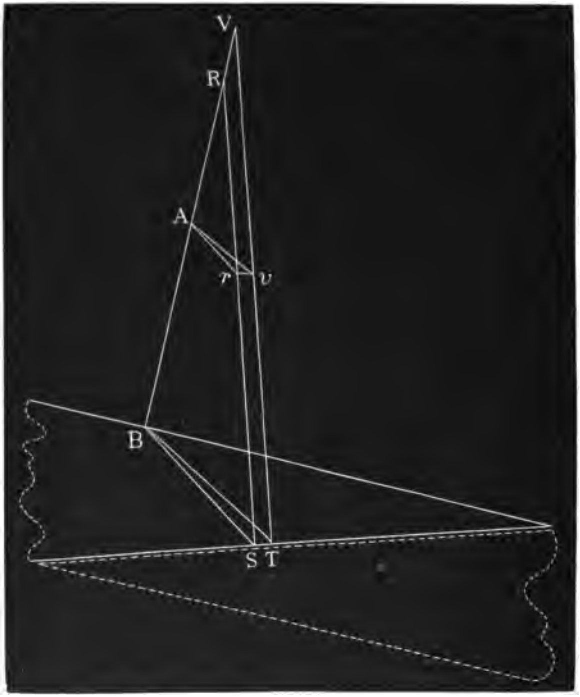

Fig. 18.

130. If A be a narrow bright line of light, parallel to the edge of the prism, it will therefore appear to be drawn out, transversely to its length, into a rectangle consisting of images of the line ranged parallel to one another, and formed, in succession, of the various homo-

geneous constituents of white light in the order of their refrangibility. If the light do not contain rays of every degree of refrangibility, some of these images will be wanting, and there will be corresponding dark lines or bands crossing this *Spectrum* (as it is called).

It is impossible, in words, accurately to describe the colours of the spectrum. From the red, which forms the less refracted terminal, it passes by insensible gradations into regions which are roughly distinguished as orange, yellow, green, blue, indigo, violet, and lavender. But these, like the great majority of colour phenomena, must be *seen*. They cannot be represented accurately even by the most accomplished artist. The chromolithographed representations, which have recently become so common, are simply beneath contempt. The amount by which any part of this spectrum is shifted from the true position of the bright slit depends (*cæteris paribus*) upon the excess of the refractive index over 1. It also depends on the angle of the prism. And, for a given angle of prism, the length of the spectrum depends upon the difference between the refractive indices for the red and the violet rays. This is called the *Dispersion*.

131. If a second prism, of the same glass, and of the same angle, as the first, be placed in a reversed position behind it (as indicated by the dotted lines in the figure), the effect of the two is simply that of a *plate* of glass with parallel faces; the rays emerge each parallel to its original direction, and there is thus no separation of colours. The reversed prism therefore simply undoes the work of the direct prism. Thus we have no dispersion, but we also have no refraction. We have, however, as has already been shown, an increase of divergence,

i.e. the image is nearer to the eye than is the object. Blair, Brewster, and Amici devised combinations of two pairs of prisms of the same glass, those of each pair having their edges parallel, such that the combination acted as a sort of achromatic telescope of low power, fitted for use as an opera-glass.

132. When we employ a prism of finite angle, we may obtain the corresponding results with great ease by

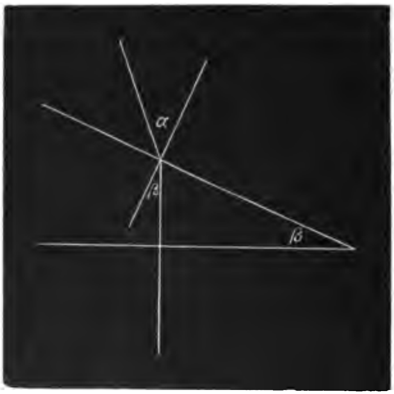

Fig. 19.

the help of the proposition in § 122, etc. Thus, let a ray fall *perpendicularly* on one face of a prism whose angle is β, fig. 19. Its angle of incidence on the other face will be β, and the angle of refraction, α, is found by

$$\sin \alpha = \mu \sin \beta.$$

Hence α is greater than β, and the escaping ray is bent *from* the edge of the prism, by an amount which is greater for greater angles of the prism (§ 123). Hence we can easily show that every ray, passing through a prism in a plane perpendicular to its edge, is (on the whole) bent from the edge of the prism. For, in either figure, 20 or 21, below, let ABCD represent the path of a ray in a plane perpendicular to the edge of the prism whose section is BEC. In BC take any point O, and through it draw PQ perpendicular to BC, meeting the sides of the prism in P and Q. BPO and OQC may now be looked upon as sections of prisms (edges, P and

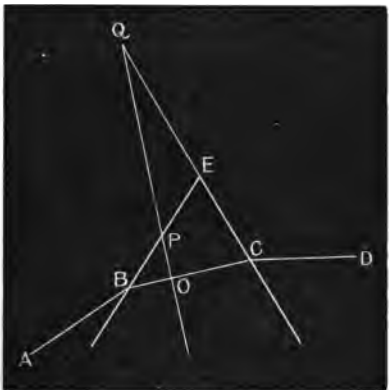

FIG. 20.

Q) such that the ray passes perpendicularly through their common face. In each the ray is bent from the edge, as just proved. When, as in fig. 20, P and Q are on the same side of BC, the sum of the angles P and Q is equal to the angle E of the original prism, and these effects are to be added. But when, as in fig. 21, P and Q are on opposite sides of BC, the angle at E is the excess of P over Q. Hence the prism P bends the ray more than Q does, and the difference is from the edge E of the original prism.

If α, β, β', α', be the successive angles of incidence and refraction, γ the angle of the prism, and δ the whole change of direction of the ray, the above figures show that

$$\delta = \alpha - \beta \pm (\alpha' - \beta'),$$

while $$\gamma = \beta \pm \beta',$$

and, of course,

$$\sin \alpha = \mu \sin \beta, \sin \alpha' = \mu \sin \beta'.$$

The upper of the two signs refers to the first case, the lower to the second. These equations enable us to find

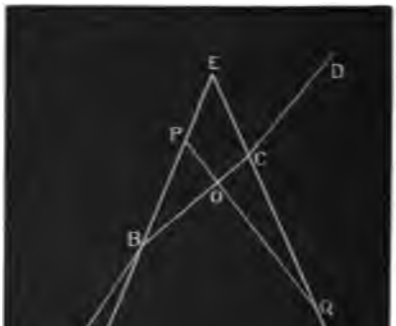

Fig. 21.

β, β', α', and δ, when α and γ are given. When all the angles are small, as was assumed in § 129 above, the last two equations become

$$\alpha = \mu\beta, \quad \alpha' = \mu\beta'.$$

Hence $$\gamma = \beta + \beta',$$

and therefore finally

$$\delta = (\mu - 1)\,\gamma,$$

which is equivalent to our former result.

133. The chief use of prisms, so far as they will be discussed here, is in spectrum analysis; and for that purpose the incident rays are carefully rendered parallel by means of a *Collimator*, simply a lens placed at its principal focal distance from the source of light, so that the rays which pass through it are deprived of divergence (§ 145). Thus we are not called upon to discuss in this work the primary or secondary focal lines due to a prism, as homogeneous parallel rays falling on it pass through, and escape from, it parallel.

134. From the fact that the angle at P (in § 122) increases faster and faster as PAC′ increases, it is clear that the deviation, by a prism, of a ray which falls perpendicularly on one face, increases faster and faster as the angle of the prism increases. Hence in fig. 20 the whole amount of deviation of the ray is greater the greater the difference between the angles BPO, OQC.

Hence for the *Minimum Deviation* as it is called, these angles must be equal, or the ray must make equal angles with the surfaces of the prism at entrance and at exit: —*i.e.* it must pass symmetrically through the prism. In this case, of course, we must use the upper signs in our formulæ, § 132, which become (as we have now $\alpha' = \alpha$ and $\beta' = \beta$)

$$2\beta = \gamma, \quad \sin\alpha = \mu \sin\beta,$$

and

$$\delta = 2\alpha - \gamma.$$

In practice γ and δ are easily measured on the limb of a divided circle to whose plane the edge of the prism is perpendicular. Thus α and β are found, and the remaining equation furnishes a simple and very accurate mode of determining μ for any species of homogeneous light, in any substance which can be formed into a transparent prism.

135. When a ray passes through a prism in a plane not perpendicular to the edge, we may obtain the direction of the refracted ray by a simple spherical projection. Let lines be drawn from the centre of the unit sphere parallel respectively to the normals to the faces of the prism, to the incident ray, the refracted ray, and the ray twice refracted. These meet the spherical surface, when all are produced towards one side of the prism, in A, B, P, Q, R respectively. The student may easily construct a figure for himself, because P, Q, A are on one great circle, R, Q, B on another; and the law of refraction gives further

$$\frac{\sin PA}{\sin QA} = \frac{\sin RB}{\sin QB} = \mu.$$

Hence R and P lie on a small circle parallel to AB, and are therefore at equal arcual distances from its pole. That is, *the incident and emergent rays are equally inclined to the edge of the prism.*

136. Newton, from some rough experiments, hastily concluded that the amount of dispersion is in all substances proportional to that of the refraction. If such were the case it is easy to see that prisms of two differently refracting materials and of correspondingly different angles, combined (as above described) so as to annul the dispersion, would likewise annul the refraction. Thus Newton was led to suppose that refraction without dispersion is unattainable.

It was found by Hall in 1733, and afterwards (independently) by Dollond, that this idea is incorrect—that, in fact, we have in certain media large refraction with comparatively small dispersion, and *vice versâ*, and thus that the dispersion may be got rid of while a part of the refraction remains. James Gregory had previously conjectured that this might be done by using, *as is done in the eye*, more media than one.

To illustrate this, we take (for certain specimens of flint and crown glass, whose optical constants were carefully measured by Fraunhofer,) the following values of the refractive index for three definite kinds of homogeneous light:—

	C	D	F
Flint glass	1·6297	1·6350	1·6483
Crown glass	1·5268	1·5296	1·5360

The rays C and F are in the red and the greenish blue regions of the spectrum respectively, and are given off by incandescent hydrogen. D is the orange-yellow ray of sodium, the source of the colour of a snapdragon flame. (See § 318.)

137. When the angle of the prism is very small (the only case we treat here), we may consider Arv (fig. 18) as approximately a straight line, whose length is (*cæteris paribus*) proportional to the angle of the prism. Also the distances Ar, Av, are to one another in the proportion of the refractive indices of red and violet rays, *each diminished by unity*. Hence, for a prism of small angle, and of flint glass such as was employed by Fraunhofer, the distances from A to its images, formed by these three kinds of homogeneous light respectively, are very nearly as

630, 635, 648.

When a prism of crown glass of the same small angle is used, they are nearly as

527, 530, 536.

The differences between the numbers for C and F are

For flint glass	18,
„ crown glass	9,

or as 2 : 1. Hence if we make the small angle of the crown-glass prism twice that of the flint, and observe A through the two prisms, with their edges turned in opposite directions, the C and F images will coincide. Both, however, will be displaced from the real direction of A as if a prism had been employed, with its edge turned as that of the crown glass was, and to the same extent as that prism would have displaced them had its refractive index been about 1·212 and the same for the two kinds of light C and F.

In fact, the displacements by the flint prism are as

630, 648,

and those by the crown prism (to the opposite side) are as

1054, 1072.

The differences, in favour of the crown prism, are as

424, 424.

But the crown prism has double the angle of the flint, so that we must write the refractive index corresponding to the prism of that double angle, which will give the same result as the combination, as

$$1 + \tfrac{1}{2}\,(0{\cdot}424), \text{ or } 1{\cdot}212,$$

as stated above.

138. This combination of prisms is called *achromatic*,

or colourless, but is not perfectly so. For if we inquire into the displacement of the D image, we find that it is as

635

for the flint prism; but as

1060

in the opposite direction, for the crown prism. Hence its whole displacement is as

425,

a little greater than that common to C and F.

The reason for this is obvious from Fraunhofer's numbers given above. The interval from C to D is to that from C to F in a greater ratio in crown than in flint glass,—so that the spectra given by these media are not *similar*. The rays of higher refrangibility are more separated in proportion to those of lower refrangibility in flint than in crown glass. This is the *Irrationality of Dispersion*—which, so far as we yet know, renders absolute achromatism unattainable. Three prisms (or lenses) in combination give a better attempt at achromatism than can be made with two; and some remarkably satisfactory results were obtained by Blair,[1] with two glass lenses enclosing a lenticular portion of a liquid. In the MS. records of the *Royal Society of Edinburgh* we find it stated that Blair exhibited, to qualified judges, a telescope of his construction which, with a focal length of twelve inches only, bore an aperture of two inches, and gave excellent definition of double stars under a magnifying power of no less than 240. Nothing approaching to this has been reached by more modern constructors.

[1] *Trans. R. S. E.*, vol. iii. (1791).

139. By looking through a prism at a very narrow slit, formed by the window shutters of a darkened room, Wollaston (in 1802) found that the light of the sky (*i.e.* sunlight) gives a spectrum which is *not continuous*. It is crossed by dark bands, indicating, as already hinted in § 130, deficiency of intensity of certain definite kinds of homogeneous light. These bands, or rather lines, were independently rediscovered, and their positions measured, by Fraunhofer[1] in 1817 with far more perfect optical apparatus. He also found similar, but not the same, deficiencies in the light from various fixed stars. The origin of these lines will be explained under *Radiation* (Chap. XVI.), along with the theory of their application in spectrum analysis. In optics they are useful to an extreme degree in enabling us to measure refractive indices with very great precision.

Wollaston's own account of his discovery is as follows:—

"If a beam of day-light be admitted into a dark room by a crevice $\frac{1}{20}$th of an inch broad, and received by the eye at the distance of 10 or 12 feet, through a prism of flint-glass, *free from veins*, held near the eye, the beam is seen to be separated into the four following colours only, red, yellowish-green, blue, and violet, in the proportions represented. . . .

"The line A that bounds the red side of the spectrum is somewhat confused, which seems in part owing to want of power in the eye to converge red light. The line B, between red and green, in a certain position of the prism is perfectly distinct; so also are D and E, the two limits of violet. But C, the limit of green and blue, is not so

[1] Gilbert's *Annalen*, lvi.

clearly marked as the rest; and there are also on each side of this limit other distinct dark lines *f* and *g*, either of which in an imperfect experiment might be mistaken for the boundary of these colours.

"The position of the prism in which the colours are most clearly divided is when the incident light makes about equal angles with two of its sides. I then found that the spaces AB, BC, CD, DE, occupied by them were nearly as the numbers 16, 23, 36, 25."[1]

140. The diagram (fig. 48) in Chap. XVI. shows, as far as mere black and white can do it, the general appearance of a small part of the solar spectrum with the chief lines, as fig. 36 shows the general aspect of the spectrum, modified by absorption. Even in the small portion shown in fig. 48, the difference of character of various lines is obvious. To exhibit all the lines which have been observed, even within the limits of the ordinary visible spectrum, on the scale of that figure, we should require a diagram many feet in length. To Kirchhoff, and subsequently to Ångström, Cornu, Smyth, and others, we are indebted for magnificent delineations of the solar spectrum, of dimensions sufficient fully to exhibit the relative

[1] "The correspondence of these lines with those of Fraunhofer I have, with some difficulty, ascertained to be as follows :—

A, B, *f*, C, *g*, D, E, . . . Wollaston's lines.
B, D, *b*, F, G, H, . . . Fraunhofer's lines.

There is no single line in Fraunhofer's drawing of the spectrum, nor is there any in the real spectrum, coincident with the line C of Wollaston's, and indeed he himself describes it as not being 'so clearly marked as the rest.' I have found, however, that this line C corresponds to a number of lines half-way between *b* and F, which, owing to the absorption of the atmosphere, are particularly visible in the light of the sky near the horizon."—Brewster, *Report on Optics, Brit. Association*, 1832.

position and magnitude of the vast series of lines observed by the help of the best modern instruments. To each of the lines in these charts is appended a mark showing the particular chemical element or compound to whose absorptive action it is due, except, of course, in the case of those lines (considerable in number) whose origin is not yet ascertained.

141. The mode of formation of a spectrum which was employed by Newton, and which is still used when the spectrum is to be seen by many spectators at a time, differs from that just explained in this, that the light from a source A (fig. 18) is allowed to pass through the prism, and to fall on a white screen at a considerable distance from it. In this case the paths of the various rays as they ultimately escape from the prism are found by joining the points $r, \ldots . v$, with the prism, and producing these lines to meet the screen. One surface of the prism must be covered by an opaque plate, with a narrow slit in it parallel to the edge of the prism, else the spectrum produced in this way is very *impure*, *i.e.* the spaces occupied by the various homogeneous rays overlap one another. To make it really pure an achromatic lens is absolutely requisite, and the incident rays should be parallel.

This leads us, naturally, to the consideration of the refraction of light at spherical surfaces.

142. Following almost exactly the same course as that taken with reflection above, let O (fig. 22) be the centre of curvature of the spherical refracting surface AB. Let U be the point-source of homogeneous light, and let PV be the prolongation (backwards) of the path pursued, after refraction, by the ray UP.

Then, *rigorously*, we have

$$\sin UPO = \mu \sin OPV,$$

where μ is the index of refraction between the two media employed. This may be written (by omitting a common factor) as

$$\frac{OU}{PU} = \mu \frac{OV}{PV}.$$

143. If, as before, the breadth of the surface be small compared with its radius of curvature, we may approxi-

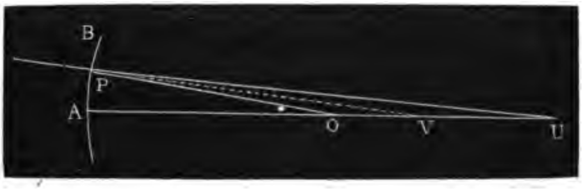

Fig. 22.

mate (sufficiently for many important practical purposes) by writing A for P. Thus we have

$$\frac{OU}{AU} = \mu \frac{OV}{AV}.$$

Retaining the same notation as in the case of reflection, we get

$$\frac{u-r}{u} = \mu \frac{v-r}{v},$$

or

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Notice that, if we put $\mu = -1$, this becomes the formula for *reflection* at a concave mirror (§ 96).

144. Suppose now that, after passing a *very short* distance into the refracting medium, the ray escapes

again into air through another spherical surface whose centre of curvature also lies in the line OA.

Let s be the new radius of curvature, w the value of the quantity corresponding to v for the escaping ray. Then, remembering that the refractive index is now $\frac{1}{\mu}$, we have (by the previous formula)

$$\frac{\frac{1}{\mu}}{w} - \frac{1}{v} = \frac{\frac{1}{\mu} - 1}{s},$$

or

$$\frac{1}{w} - \frac{\mu}{v} = \frac{1-\mu}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

Adding (1) and (2) we get rid of v (which indicates the behaviour of the rays in the substance of the lens) and have

$$\frac{1}{w} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

This contains the whole (approximate) theory of the behaviour of a very thin *Lens.*

145. When the source is at an infinite distance, or $u = \infty$, we have

$$\frac{1}{w} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$
$$= \frac{1}{f} \text{ suppose.}$$

This quantity f, defined entirely in terms of the refractive index and of the curvatures of the two faces of the lens, is called the *principal focal distance.*

If μ be greater than 1, *i.e.* as in the case of a glass lens in air, f is *positive* if

$$\frac{1}{r} - \frac{1}{s}$$

be so; and it obviously retains the same value, and sign, if the lens be turned round. For, in the formula, r and s change places, and they also change signs; *i.e.* we must put $-s$ for r and $-r$ for s. This leaves the result unchanged.

146. All lenses, therefore (of substances having a greater refractive index than air), whose sections are of any of the forms in fig. 23, whichever way they are

FIG. 23.

turned, render parallel rays which pass through them *divergent*. Their characteristic is that they are thinnest at the middle.

But the expression

$$\frac{1}{r} - \frac{1}{s}$$

is negative for lenses whose sections are of any of the forms shown in fig. 24. Such lenses, whichever way they are turned, render parallel rays *convergent*. Their characteristic is that they are thickest at the middle.

But these characters are interchanged when μ is less than 1; as, for instance, when the lens is an air-space surrounded by water.

The similarity on reversal is *not* in general true in a second approximation.

147. The formula for a thin lens now takes the form,

$$\frac{1}{w}-\frac{1}{u}=\frac{1}{f},$$

and differs from that for a curved reflecting surface only in the sign of the second term. With the proper

FIG. 24.

allowance for this, then, all that we have said of direct reflection at spherical mirrors holds true of direct refraction through thin lenses with spherical surfaces.

We may now put the whole matter in the excessively simple form which follows:—

A thin lens increases or diminishes by a definite quantity the convergence or divergence of all rays which pass directly through it.

This quantity is the divergence or convergence of rays falling on the lens from, or passing from it to, its principal focus. Or it is the convergence or divergence which the lens produces in parallel rays.

Thus, if the distance of an object from a convex lens is twice the focal length of the lens, the image is formed at the same distance from the lens on the opposite side, and is equal in size to the object.

148. Figs. 25 and 26 show the production of a real

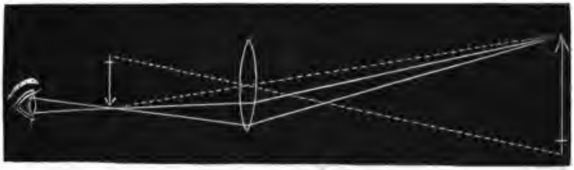

FIG. 25.

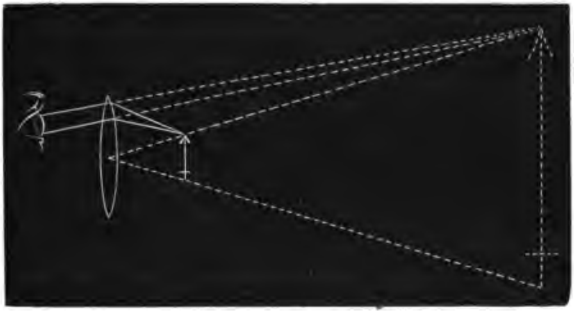

FIG. 26.

image and of a virtual image by lenses which produce *convergence* of parallel rays—along with the rays by which one point of each is seen by an eye placed behind the lens. In either case it is obvious from the cut that the sizes of object and image are, respectively, as their distances from the centre of the lens.

Fig. 25 shows how such a lens produces a *real* inverted image of a body placed farther from it than its principal focus. This is the case in the camera obscura, in the solar microscope, and in the object-glass of a telescope.

Fig. 26 shows how a *virtual* erect image is formed of a body placed nearer to a lens than its principal focus. This is the case of a single lens used as a microscope.

In the former case the divergence of the incident rays is so small that the lens renders them convergent.

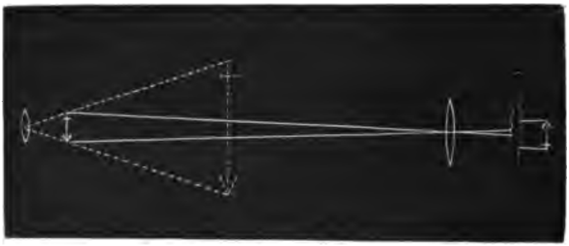

FIG. 27.

In the latter the divergence is so great that the lens can only diminish, not destroy it. In using a hand-magnifier in this latter way, we so adjust it, by practice, that the enlarged image appears to be formed at the distance from the eye at which vision is most distinct. It is obvious that the amount of magnification must, then, be greater as the focal length of the lens is less.

149. We can now understand the working of the ordinary *astronomical telescope* (fig. 27). The object-glass furnishes an inverted but real image of a distant body, *within our reach.* We can, therefore, place the eye-glass (like the single microscope above spoken of) so as to

form a virtual magnified image of this real image, treated as an object. It is still, of course, inverted. It is easy to see that the angle, subtended at the eye by the virtual image seen through the eye-glass, is to that subtended by the object at the unaided eye, approximately as the focal length of the object-lens is to that of the eye-lens. These angles are, in fact, those subtended at the centres of the two lenses by the real image. This ratio is, therefore, called the *magnifying power* of the telescope.

150. The *compound microscope*, in its simplest form, is precisely the same arrangement as the astronomical telescope. The only difference is that the object, being at hand, can be placed near to the object-glass (still, however, beyond its principal focus), so that the real image formed is already considerably larger than the object, and is then still farther magnified by the eye-glass.

151. The magnifying power of a single lens, when used as a hand microscope, is to be measured by the ratio of the angle under which the virtual image of an object is seen (at the distance of most distinct vision) to that at which the object itself would be seen (at that same distance); *i.e.* it is the ratio of 10 inches to the focal length of the lens.

152. From the formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)$$

we see that the focal length of a simple lens is less as μ is greater. Thus all that we have just said is true only for homogeneous light.

But if we combine two thin lenses, placing them close together, we may arrive at an approximately achromatic arrangement. For we have, for the first lens

$$\frac{1}{w}-\frac{1}{u}=\frac{1}{f}.$$

For a second, close to it, we have

$$\frac{1}{x}-\frac{1}{w}=\frac{1}{f'}.$$

For the two, considered as one, we have

$$\frac{1}{x}-\frac{1}{u}=\frac{1}{f}+\frac{1}{f'}=\frac{1}{f''},$$

where f'' is the focal length of the combination.

Now

$$\frac{1}{f}+\frac{1}{f'}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)+(\mu'-1)\left(\frac{1}{r'}-\frac{1}{s'}\right),$$

and there is an infinite number of ways in which r' and s' can be chosen, even when r and s are given, such that the value of the right-hand side shall be the same for each of *two* values of μ and the corresponding values of μ'.

Any one of these gives an achromatic combination, of the necessarily imperfect kind described above in considering prisms (§ 138).

But, as we have now the curvatures of *four* surfaces to deal with, we can adjust these so as not only to make the best attainable approximation to achromatism, but also to reduce the unavoidable spherical aberration to a minimum.

153. These questions, however, are beyond the scope of this work. We can remark only that the adjustment for two rays, for which the refractive indices are μ and

$\mu+\delta\mu$ in the first medium, and μ' and $\mu'+\delta\mu'$ in the second, requires the *one relation*

$$\frac{\delta\mu}{\mu-1}\frac{1}{f}+\frac{\delta\mu'}{\mu'-1}\frac{1}{f'}=0,$$

which involves only the ratio of the focal lengths of the two lenses—leaving their forms absolutely undetermined. But, if both μ and μ' be greater than unity, the signs of f and f' must be different;—*i.e.* in an achromatic combination of two lenses one must be convex and the other concave.

The reader must, however, be reminded that we are dealing with a first approximation only, and that spherical aberration does not come in till we reach a second. The details for a proper achromatic combination must be sought in special treatises on theoretical optics.

154. Before leaving this subject, however, we must find the behaviour of two thin lenses which are placed at a finite distance from one another.

For the first lens we have, as before,

$$\frac{1}{w}-\frac{1}{u}=\frac{1}{f}.$$

If the second lens be placed at a distance a behind the first, the rays which fall on it appear to come from a distance $w+a$. Hence, for the light emerging from the second lens, we have

$$\frac{1}{x}-\frac{1}{w+a}=\frac{1}{f'}.$$

When u is infinite, we have from the last two equations

$$\frac{1}{x}=\frac{1}{f+a}+\frac{1}{f'}.$$

It is obvious that a combination of this nature offers the same kind of facilities for the partial cure of dispersion and of spherical aberration as when the lenses are in contact, with *one* additional disposable constant. Thus we are enabled to construct compound *achromatic eye-pieces*, which can be corrected for spherical aberration also.

155. We may now go back to the formation of an image by a prism, and inquire how, by the use of an achromatic lens, we can project a pure spectrum on a screen (§ 141).

We have seen that a thin prism, for rays falling nearly perpendicular to it, forms a virtual and approximately rectilineal image of a luminous point, in which the colours are ranged in order of refrangibility.

Suppose the light which passes through the prism to fall on an achromatic lens, placed at a distance greater than its focal length from the virtual image above mentioned. These rays after passing through the lens will proceed to form, at the proper distance, a real linear coloured image of the luminous point, in which (as in the virtual image) the colours do not overlap.

Instead of a luminous point, rays diverging from a very narrow slit parallel to the edge of the prism are employed. It is usual to place the lens at *double* its focal distance from the virtual image, and thus the real image is formed at an equal distance on the other side of it, and is of the same size as the virtual image. It may now, if required, be magnified by means of an achromatic eye-piece. Or, in other words, it may be examined by means of a telescope.

In fact a telescope, whose object-glass is covered by a thin prism, has been usefully employed during a total eclipse in examining the light of the solar corona.

A similar arrangement, made to have an exceptionally large field of view, is employed to find the nature of the spectra of meteorites or falling stars.

156. A very simple but interesting case of refraction at a cylindrical surface is furnished by a thermometer tube. It is easily seen that the diameter of the bore appears, to an eye at a distance, large compared with the diameter of the tube, to be greater than it really is, and in the proportion of the refractive index of the glass to unity. Thus in flint glass it appears magnified in about the ratio 5 : 3. Hence the mercury appears completely to fill the external surface of such a tube, if the bore be only $\frac{3}{5}$ths of the external diameter.

CHAPTER X.

REFRACTION (*continued*)—CAUSTICS.

157. BUT a far more interesting case is that of parallel rays falling on a full cylinder of glass or water. Its interest consists in the fact that by its aid we can explain the chief phenomena of the *Rainbow*, which is one of the most interesting of the whole family of caustics. We, accordingly, devote special attention to it.

The problem, without losing any of its applicability to the rainbow, is much simplified by supposing the rays to be incident in a direction perpendicular to the axis of the cylinder; for in this case the whole course of each ray is in a plane perpendicular to the axis. We need not treat here of rays which pass *close to* the axis of the cylinder. For such the cylinder acts as a lens, and its focal length (to the usual first approximation) can easily be obtained by methods such as those given above. What we are mainly concerned with is the behaviour of the rays which escape into the air, after *one*, or *two*, reflections at the inner surface of the cylinder.

Suppose first that we consider a ray once reflected in the interior of the cylinder. Let SP (fig. 28) be one of the set of incident parallel rays, and let its path be SPQP′S′. This involves refraction at P, reflection at Q, and again refraction at P′. But it is obvious from the symmetry of the circular section, and from the laws of refraction and reflection, that this path is symmetrical about the line OQ which joins the axis of the cylinder to the point at which the ray is reflected. Hence SP, S′P′ meet OQ in the common point *s*; and the amount by which the direction of the ray has been turned round by the refractions and the reflection is twice the supplement of half the angle at *s*. But the angle POR is double of OPQ, the angle of refraction, while OP*s* is equal to the

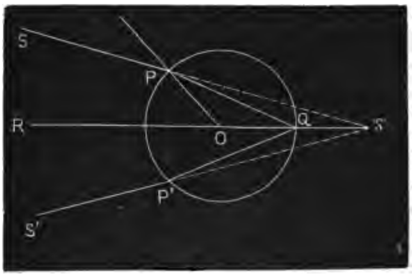

FIG. 28.

angle of incidence. Hence the half angle at *s* is the excess of twice the angle of refraction over the angle of incidence.

But, in § 125, we showed that this quantity has a maximum value. Now one of the conditions of a maximum or minimum of any quantity is that, near it, the value of the quantity *changes very slowly*. Thus a number of issuing rays are crowded together near the direction corresponding to this maximum, the others being more widely scattered,—while for all of them the angle at *s* is smaller.

Newton gives us as an illustration of this, the very slow change of length of the day when the sun is near one of the tropics.

158. We may, for comparison with what follows, give another mode of investigating this maximum, though the elementary process already given is applicable at once to rainbows of all orders. If θ be the angle of incidence, ϕ that of refraction, and μ the refractive index, we have to find the maximum value of

$$\tfrac{1}{2}s = 2\phi - \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

with the condition furnished by the law of refraction

$$\sin\theta = \mu \sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

These give at once

$$2d\phi = d\theta\,,$$

and

$$\mu \cos\phi\, d\phi = \cos\theta\, d\theta.$$

Hence

$$\mu \cos\phi = 2\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

From (2) and (3) we have, as in § 125 above,

$$3\cos^2\theta = \mu^2 - 1\,;$$

which determines the requisite angle of incidence. The values of the other quantities are easily calculated from this; and we finally have, for the maximum value of the sine of the half angle at s, the expression

$$\frac{1}{\mu^2}\left(\frac{4-\mu^2}{3}\right)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

This is obviously smaller as μ is greater, at least up to the limit $\mu = 2$.

With the value $\frac{4}{3}$ for μ (which is nearly that for yellow rays refracted into water) we have

$$\text{L} \sin \tfrac{1}{2}s = 9{\cdot}55462,$$

which corresponds very nearly to

$$\tfrac{1}{2}s = 21^\circ\ 1'.$$

159. Now suppose the diameter of the cylinder to be small compared with the distance of the eye from it. In this case the point s may be considered as being in the axis of the cylinder.

Let SsE_1 (fig. 29) be made equal to the maximum value of s; then an eye placed anywhere in the line sE_1 will receive the rays which are congregated towards the

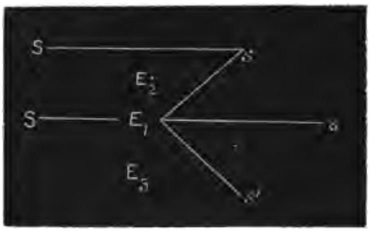

FIG. 29.

maximum. An eye *within* the angle SsE_1 (as at E_2) will receive some of the straggling rays, while an eye outside that angle (as at E_3) will see nothing.

Let there now be imagined a great number of parallel cylinders; let $E_1\sigma$ be drawn parallel to the incident rays, and make the angle $\sigma E_1 s'$ equal to $\sigma E_1 s$. Then the eye at E_1 will see the concentrated rays (already spoken of) in the directions $E_1 s$ and $E_1 s'$. From points within $sE_1 s'$ some straggling rays will reach it, from points outside that angle none.

160. Now suppose cylinders to be placed in great numbers *in all directions* perpendicular to the incident rays. The eye at E_1 will see a bright *circle* of light

whose centre is in $E_1\sigma$. Inside that circle there will be feeble illumination; outside it, darkness.

This is obviously the case of the rainbow, where we have spherical drops of water instead of the cylinders above spoken of. For each spherical drop is effective only in virtue of a section through its centre, containing the incident ray and the eye; and such sections are the same as those of the cylinders.

161. Thus far we have been dealing with *parallel* rays of *homogeneous* light; and the appearance (to the degree of approximation we have adopted) is that of a bright circle whose centre is diametrically opposite to the source of light, whose radius is (for raindrops) about 42° 2′, and whose area is slightly illuminated.

Introduce the idea of the different kinds of homogeneous light which make up sunlight, and we find a circular (almost pure) spectrum—the less refrangible rays being on the outside.

Next introduce the consideration of the finite disk of the sun, and we have an infinite series of such arrangements superposed on one another, the centre of each individual of the series being at the point diametrically opposite to the point of the sun's disk which produces it. This leaves the *general* aspect of the phenomenon unchanged, but altogether destroys the purity of the spectrum.

These results are in *fair* accordance with the phenomena of the principal, or *primary*, rainbow. We shall presently find why the coincidence is not perfect.

162. If we next consider light which has been twice reflected within the cylinder, we have an arrangement like the diagram fig. 30; where the lettering is as nearly as

possible the same as that in fig. 29. Everything is still symmetrical about the line Os, which obviously cuts at right angles the ray QQ′.

Reasoning precisely similar to that above given shows that the complement of half the angle at s is now equal to the excess of *thrice* the angle of refraction (OPQ) over that of incidence (the supplement of OPs), and that this

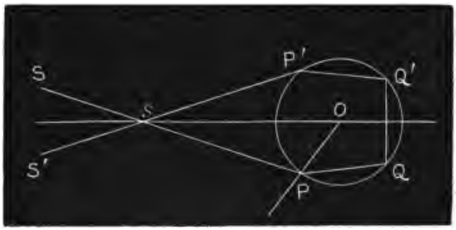

FIG. 30.

also has a maximum value, *i.e.* that s itself has a minimum value.

163. To find this minimum, we may proceed as in § 125, or as in § 158. Taking the latter method, we have

$$\frac{\pi}{2} - \tfrac{1}{2}s = 3\phi - \theta,$$

with

$$\sin\theta = \mu \sin\phi.$$

Differentiating, etc., as before, we find

$$8 \cos^2\theta = \mu^2 - 1,$$

whence, finally,

$$\sin \tfrac{1}{2}s = \frac{\mu^4 + 18\mu^2 - 27}{8\mu^3}.$$

This quantity increases with μ, for its differential coefficient is

$$\frac{1}{8}\left(\frac{9}{\mu^2} - 1\right)^2,$$

which is necessarily positive. (It vanishes, no doubt, for $\mu=3$, but then so does θ.)

For $\mu=\frac{4}{3}$ the value of $\sin\frac{1}{2}s$ is

$$0{\cdot}4303 \text{ nearly},$$

so that

$$s=50^\circ\ 58'.$$

164. Carrying out the same steps of reasoning as before, and applying the result to raindrops, we find a second rainbow concentric with the first, but with a greater radius, viz., about 51° (for yellow light). All the above remarks about the impurity of the spectrum, etc., apply to this bow also.

In this bow the less refrangible rays are on the *inner* side, and the straggling rays illuminate feebly the space *outside* it. Hence the space between the red boundaries of the two bows has no illumination from rays reflected either once or twice within the water-drops.

We might next consider the rainbows due to two refractions and three, four, or more internal reflections. But these are too feeble to be observed.

165. A great deal of futile discussion has been raised by the question, "Can two spectators see the same rainbow?" The simplest consideration of the essential nature of a caustic shows that such a question has no meaning, if the words are all taken in their usual sense. We might as well ask the question, "Can one see the same rainbow with each eye?"

Another question of the same kind is, "Can a rainbow be seen by reflection in still water?" To this, of course, the answer is that a spectator sees, by reflection in still water, the rainbow he would have seen had the water been removed, and had his eye been at the position formerly occupied by its image in the water. But a reflected rainbow differs from a rainbow seen directly, in the fact that, as the light forming the latter is partially *polarised*, the intensity of the former is modified differently at different points in the act of reflection. This, however, belongs to Physical Optics.

Intersecting rainbows are not uncommon. They require, of course, for their production, *two* sources of parallel rays; and they are seen when, behind the spectator, there is a large sheet of calm water. The usual rainbows are furnished by the direct sunlight, and a new series (never so brilliant, though of exactly the same character and dimensions) are formed by the light coming, as it were, from the image of the sun in the water. As this image is always as much below the horizon as the sun is above it, the centre of the new system is as much above the horizon as that of the ordinary system is below it.

166. What we have now given is nearly all that geometrical optics can tell us about the rainbow.

It seems that the first really important steps in the explanation, viz. (1) that the primary bow is due to rays falling on the outer portions of the drops, which suffer two refractions and one reflection before reaching the eye, and (2) that the secondary bow is due to rays falling on the inner side, and suffering two refractions and two reflections, are due to Theodorich, about 1311. His work was not published, and its contents were first announced by Venturi[1] in the present century. These

[1] *Commentari sopra la storia e le teorie dell' Ottica*, Bologna, 1814.

results were independently discovered by De Dominis[1] in 1611.

Neither of these writers, however, pointed out the concentration of the rays in particular directions. This was done by Descartes in 1637, by the help of Snell's law. He calculated with great labour the paths of each of 10,000 parallel rays falling on different parts of one side of the drop, and showed that from the 8500th to the 8600th the angle between the extreme issuing rays is measured in *minutes* of arc,—thus discovering by sheer arithmetic the maximum which, as we have seen above, is so easily found by less laborious methods.

Newton's addition to this theory consisted mainly in applying his discovery of the different refrangibilities of the different homogeneous rays. The explanation was then thought to be complete. For a long time this was held to be one of Newton's most brilliant discoveries. It is well to notice that he himself speaks of it in its true relation to the work of his predecessors. He merely says:—"But whilst they understood not the true origin of colours, it is necessary to pursue it here a *little further.*"

And he said well; for a full investigation, conducted on the principles of the undulatory theory, introduces, as was first pointed out by Young, certain important modifications in the above statements. Of these we need mention only two, viz. that in each bow there is more than one maximum of brightness for each homogeneous ray, and that the principal maximum, which gives the ordinary primary bow, is of somewhat less angular diameter than that assigned by geometrical optics.

[1] Newton, in his *Optics*, says the work of De Dominis was written twenty years before it was published.

The *spurious* bows, as they are called, which often appear like ripples, inside the primary and outside the secondary bow, and which depend upon the other and fainter maxima just mentioned, have no place even in Newton's theory. About them, in fact, geometrical optics has nothing to say. Young, in 1803, took the first step for their explanation. They were fully investigated, from the undulatory point of view, by Airy, in 1836-38; and his results were completely verified by the measurements of Hallows Miller in 1841.[1] Miller used a fine cylinder of water escaping vertically from a can, as suggested by Babinet. This is *one* of the reasons which made us treat the case as one of refraction and reflection in a right cylinder. The other will appear immediately.

167. The overlapping of the colours in the rainbow is occasionally so greatly exaggerated that only faint traces of colour appear.

This may happen when the sun shines on raindrops in the lower strata of the atmosphere through thin clouds in the higher strata. Thus the effective source of light is virtually spread over a much larger spherical angle, and there is no sharp edge to it as in the case of the unclouded disk. In fact, the sky, for a degree or two round the sun, often becomes dazzlingly bright from this cause. The rainbow is then much broader and fainter than usual, and nearly white.[2]

But, besides this, the smaller the size of the rain-drops the greater are the modifications produced (§ 166) in the

[1] Airy's paper is in vol. vi. of the *Cambridge Phil. Trans.;* Miller's in vol. vii.

[2] See *Proc. R. S. E.*, ix. 542, for a description, by Sir R. Christison and others, of a singularly definite occurrence of a white rainbow.

results given by the geometrical theory. Thus, even when the sun is unclouded, if there be a mixture of rain-drops of very different sizes, there will be a *superposition* of sets of true and spurious bows of different diameters. Hence another possible source of a white rainbow.

When the moon is the source of light, the rainbow is so faint that it is often difficult to distinguish the colours; but with full moon, and other favourable circumstances, it is easy to assure one's self that the colours are really present. It is certain that lunar rainbows must occur with the same average frequency as solar rainbows; but their faintness, and the fact that they occur at night, both tend to make them much more rarely seen.

Rainbows must not be confounded with *Glories*, as they are called, coloured circles of small radius which are often seen surrounding the shadow of an observer when it is cast on mountain mist (the *Brocken-Spectre*). These are, according to Young (*Lectures*, II., 645), probably analogous to the colours of thin plates; and therefore depend on the size of the water particles. That these particles are often of extraordinary uniformity in size, is proved by the frequent appearance of coronæ (§ 172). This explanation is by no means certain. It seems possible that glories may be due to a cause somewhat analogous to that which produces the spurious rainbows. The calculations requisite to settle this question present considerable labour, but no great difficulty.

168. The refraction of sunlight or moonlight, through ice-crystals forming cirrus clouds, gives rise to coloured *Halos*, *Parhelia*, *Paraselenæ*, etc. Halos are at once distinguished from rainbows; for they *surround* the

luminary, while the primary and second rainbows, at least, have their centres *opposite* to it.

We must confine our remarks to the more common of these phenomena, and this is of the less consequence as the others are very rarely seen except in high latitudes.

The commonest forms of ice-crystals in the air are regular hexagonal prisms, terminated by plane ends perpendicular to the axis. Sometimes the prisms are long and narrow, sometimes they are mere hexagonal plates. Two alternate faces of the hexagon give an ice-prism of 60°, a face and an end a prism of 90°. The refractive index of ice is about 1·31. Let us study the effects of innumerable prisms having these angles, and falling in all aspects or positions in the quarter where the sun appears to be situated. The result of an average distribution will, of course, be symmetrical with regard to the line joining the eye and the sun. Also, as in § 157, the refracted rays will be crowded together in the directions of minimum deviation.

169. Calculating by the method of § 134, we find, for the minimum deviation produced by an ice-prism of 60°, the angle 22° very nearly; and, for a prism of 90°, about 46°. This is, of course, on the supposition that the refraction is in a plane perpendicular to the edge of the prism.

Hence, if we consider one kind of homogeneous light only, the sun should appear to be surrounded by two circular rings of light, each of a breadth equal to the sun's apparent diameter, and of mean angular radii 22° and 46° respectively. As these are due to the minimum of deviation by each class of prisms, the scattered rays directly refracted by any one of the prisms, as well as the rays which have passed obliquely through it, are seen *outside* the corresponding halo.

The minimum deviation will, of course, be least for the least refrangible rays, so that in both these halos the red rays form the *interior* border. In this respect they resemble the secondary, not the primary, rainbow. Otherwise the remarks made above with regard to the impurity, etc., of the colours of the rainbow apply, with even greater emphasis, to the halos. In fact, only the red is at all pure, and the overlapping (due partly to the sun's diameter, but still more to oblique refraction) is so great that, as a rule, a mere trace of green and blue can be seen, the external portion of each halo being nearly white. This gives the phenomenon a very singular character.

170. Next let us take account of the fact that the crystals free in air will, on the whole, tend to fall in one or more particular positions, *i.e.* the long prisms mainly endwise, or it may be with their axes horizontal; the plates flatwise, or edgewise, as the case may be.

The effect will, of course, be to intensify those parts of the halo which are due to the majority of the crystals. When the sun is near the horizon, and the vertical ice-prisms exceptionally numerous, the parts of the halo of 22° which are at the same level as the sun are coloured spectra, sometimes dazzlingly bright, and they are called *Parhelia*, or, vulgarly, *Mock-Suns.*

When the sun is *not* on the horizon, the paths of the rays through the vertical prisms are no longer in planes perpendicular to their edges. Here also there is a position of minimum deviation, but the deviation is greater than before, increasing with the obliquity of the rays. Also a ray, passing anyhow through a prism, makes equal angles with the edge (in this case vertical) before and after passing (§ 135). This is evident, without calculation, from the principle of reversal. Hence, as the sun rises higher, the parhelia gradually separate outwards from the halo, still keeping, however, at the same apparent altitude as the sun.

171. If there be an excess of hexagonal prisms, or plates, with their axes horizontal, these also will produce parhelia, which will be situated on the halo above and below the sun, if the axes of the prisms are perpendicular to the line joining the sun and the spectator. But as they are as likely to lie in any other (horizontal) direction, there will be a continuous series of parhelia, forming a new halo, which *touches* that of 22° externally above and below, and may even, under favourable circumstances (such as the requisite altitude of the sun, etc.), assume a complete quasi-elliptic form.[1] This halo is brightest at its upper and lower portions, which are usually the only parts visible, and they are therefore commonly called the *tangent-arcs* to the halo of 22°.

Similar remarks apply to the halo of 46°. And the hexagonal pyramids, which sometimes terminate the ice-prisms, produce analogous results. The parhelia, also, are sometimes themselves bright enough to produce secondary halos; and the *reflection* of the sun's rays from the surfaces of the crystals gives, from an excess of horizontal prisms, a colourless, vertical, great circle passing through the sun; from an excess of vertical faces, a colourless, horizontal, small circle at the same altitude as the sun.

[1] The writer was fortunate enough to see this phonomenon (nearly complete) on May 10, 1876. See *Proc. R. S. E.*, ix. 425.

All of the features which we have described, with the exception of the colourless vertical great circle, will be easily recognised in fig. 31, which is reduced from a

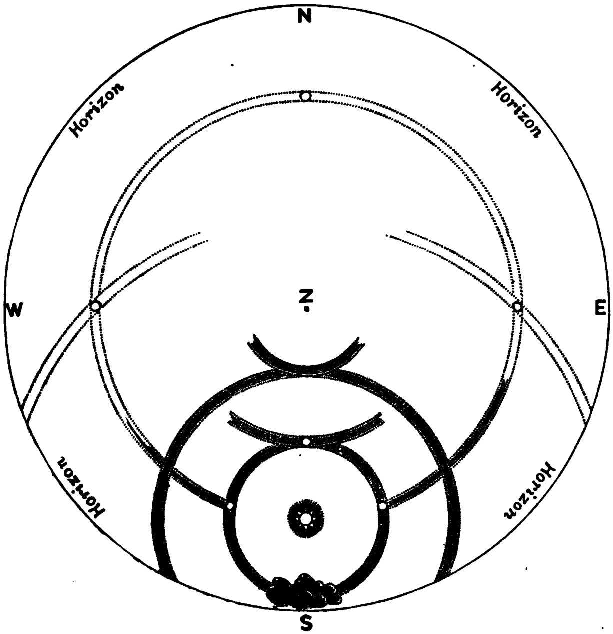

FIG. 31.

drawing by Hevelius[1] of a remarkable series of Halos and Mock-Suns seen by him at Ghent in 1661. There is, however, one feature in this drawing, viz. the whitish, incomplete circle of about 90° radius surround-

[1] *Mercurius in Sole visus.* Gedani, 1662. The passage is quoted in full by Huygens in his treatise *De Coronis et Parheliis,* Leyden, 1703.

ing the sun, which has been observed by others than Hevelius, but is not yet explained.

For full details on this very interesting subject we must refer to the remarkable memoir by Bravais,[1] who studied the phenomena in high latitudes. The *general* explanation of the production of Halos was suggested by Mariotte, but was first accurately given by Young.

172. Halos must not be confounded with *Coronæ*—those concentric rings which encircle the sun or moon when seen through a mist or cloud. Halos, as we have seen, are red *inside*, coronæ are red *outside*. Halos have definite radii depending on the definite angles of ice-crystals; the size of the coronæ, whose radii are as 1 : 2 : 3, etc., depends on the size of the drops of water in a mist or cloud, being smaller as the drops are larger. Thus their diminution in radius shows that the drops are becoming larger, and implies approaching rain. They are due to *Diffraction*, and can only be explained by the help of the undulatory theory.

[1] *Journal de l'École Polytechnique*, xviii. 1847

CHAPTER XI.

REFRACTION IN A NON-HOMOGENEOUS MEDIUM.

173. THE principles already explained are sufficient for the purpose of treating this question also. But they require for their application the artifice of supposing the medium to be made up of layers, in each of which the refractive index is the same throughout the layer, but finitely differs from one layer to another; and then supposing these layers to become infinitely thin and infinitely numerous. In this case there will, of course, be only an infinitely small difference in properties between contiguous layers; and the abrupt change of direction which accompanies ordinary refraction is now replaced by a *continuous curvature* of the path of the ray.

174. Glimpses of a more general method had been obtained even in the seventeenth century; and in the eighteenth these had become a consistent process so far as application to the corpuscular theory is concerned.

In fact the problem of the motion of a corpuscle is merely a case of the ordinary *Kinetics of a Particle.* Newton, as we saw in § 128, gave the complete solution of the problem of refraction at a plane surface, from this point of view; and the study of the motion of a corpuscle in a non-homogeneous isotropic medium is merely that of the motion of a material particle in a region such that its speed in passing through any point of that region depends upon the co-ordinates of the point alone.

The region may be conceived to be intersected by surfaces, every point of any one of which corresponds to the same definite speed. [These are the surfaces bounding the above-described layers of uniform refractive index; for, on this theory, the refractive index is proportional to the speed of the corpuscle.]

The acceleration of the motion of the corpuscle must therefore be at every point perpendicular to the surface of this class on which it lies at the moment; and its amount is measured by the rate at which the kinetic energy increases per unit of length in the direction of the normal to that surface.

Hence, knowing the direction of motion and the speed of the corpuscle in any position, we can combine with this information our knowledge of the direction and magnitude of the acceleration, and the proper mathematical methods enable us to find the form of the path, *i.e.* of the *ray*.

175. Thus, by the ordinary results of kinetics, we see that:—

(1) *The osculating plane (or plane of bending) of the path contains the normal to the surface of equal speed through which the corpuscle is passing;*

(2) *The so-called "centrifugal force" is balanced by the acceleration along the normal to the path.*

Since the refractive index, μ, is proportional to the speed, this condition (2) is expressed analytically by

$$\frac{\mu^2}{\rho} = \frac{d}{dn}(\tfrac{1}{2}\mu^2),$$

or

$$\frac{1}{\rho} = \frac{1}{\mu}\frac{d\mu}{dn},$$

where ρ is the radius of curvature of the path, and n is measured along that radius of curvature.

176. As a first, and simple example, let us take ordinary terrestrial refraction, as depending on the density and temperature of the atmosphere at different heights. To simplify matters we may suppose the earth's surface to be *plane*, and the rays considered to be so nearly horizontal that the direction in which the refractive index changes most rapidly (*i.e.* the vertical) is practically everywhere at right angles to the ray. This implies that, as is the case in still air, the refractive index depends only on the height above the earth's surface. When the general nature of the phenomenon is fully understood from this point of view, there is no difficulty in seeing what are the modifications which are due to the earth's curvature.

The refractive index of air at 0° C., and 760^{mm} pressure (hereafter denoted by Π) is about

$$1\cdot000294, \text{ or } 1 + \frac{1}{3400};$$

and, for other temperatures and pressures, the excess over unity (the *Refractive Power*, as it is called) varies directly as the density. Hence, at temperature t and pressure p, it is

$$1 + \frac{p}{3400\Pi}\,\frac{274}{274+t}.$$

When the temperature is assumed to be constant, say zero, through a layer of a few hundred feet thick, the value of p at a height x feet above the surface is approximately

$$p=\Pi\left(1-\frac{x}{26,200}\right),$$

where the divisor of x is the so-called "height of the homogeneous atmosphere" for 0° C.

Thus the equation of § 175 shows that the concavity of a nearly horizontal ray is, in such an atmosphere, *downwards*, and the curvature (to the foot as unit) is

$$\frac{1}{3400\times 26,200}.$$

To the same unit, the curvature of a meridian of the earth is (approximately)

$$\frac{1}{4000\times 5280},$$

which is nearly four times as great.

If we suppose the temperature of the air to vary with elevation, we obtain to a sufficient approximation

$$\frac{1}{\rho}=\frac{1}{3400}\left(\frac{1}{26,200}+\frac{1}{274}\frac{dt}{dx}\right)$$

Hence there is no curvature, *i.e.* no difference of refractive index, if

$$\frac{dt}{dx}=-\frac{274}{26,200};$$

that is, if the temperature fall by 1°·05 C. for every hundred feet of ascent. In this case the lower air has, throughout, the same density. If the temperature fell off at a greater rate than the above, the upper air would be denser than the lower, and we should have instability.

If, on the other hand, there were a uniform *rise* of temperature of (say) 1° C. per 10 feet of ascent (a possible, but very unusual occurrence) we should have, for the curvature of a nearly horizontal ray,

$$\frac{1}{\rho}=\frac{1}{3400}\left(\frac{1}{26,200}+\frac{1}{2740}\right),$$

about $2\frac{1}{2}$ times the curvature of the earth's meridian.

It is with quantities lying between these narrow

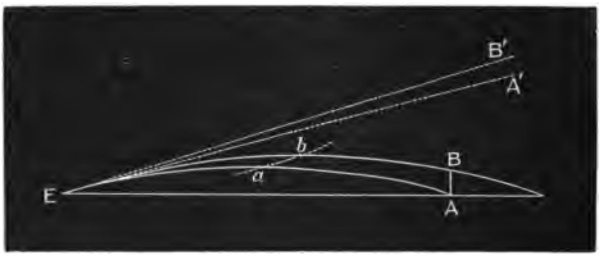

Fig. 32.

limits that we have to deal when we try to explain the phenomena of *Mirage*, to which we proceed.

177. We continue, as in the preceding section, to suppose that the density, and therefore the refractive index, depends only on the height above the earth's surface. Thus every ray which, leaving the surface, returns to it again must, by the principle of reversal, have a *symmetrical* form, and therefore a vertex (or highest point) midway between its terminals.

If the rate of change of temperature per vertical foot be uniform, the rays to which we limit our consideration

(§ 176) will be arcs of *equal* circles. These, of course, do not intersect, and thus the image of an object AB (fig. 32) appears to an eye at E in some such *erect* position as A′B′ in the figure, EA′ and EB′ being tangents at E to the rays EA and EB respectively. This is the ordinary terrestrial refraction, and will obviously be greater as the rate of diminution of density per foot upwards, in the air, is greater. Vince records that at Ramsgate, on certain occasions, he saw from the water's edge the cliffs near Calais, which in clear ordinary states of the atmosphere are "frequently not visible from the highlands about that place."

FIG. 33.

Observe particularly that, in this case, the line joining the vertices *a*, *b*, of successive rays slopes upwards *from* the spectator.

But next suppose the rate of diminution of density to be small (or even *nil*) in the layer of 50 or 100 feet thick, nearest the earth, but to become greater in the layer just above that. The state of things is represented by the annexed diagram (Fig. 33).

The image of the object AB is now seen from E in some such position as B′A′, *i.e.* it is elevated and *inverted*. At the same time the object will be seen directly, through the lower nearly uniform air.

Observe particularly that, in this case, in which the rays cross one another on their way from the object to the eye, the line joining the vertices, b, a, slopes upwards *towards* the spectator.

178. Hence, given a distribution of density (in horizontal layers), we can at once tell the number and nature of the images of a distant object on the horizon, say a ship at sea, which will be produced; provided we calculate the form of the curve on which lie the *vertices* of all rays leaving the eye in the vertical plane containing the object. Midway between eye and object erect a perpendicular to the earth's surface. Each intersection of this line with the curve is the vertex of a ray by which the object can be seen. If the curve of vertices at one of these intersections slope upwards *from* the eye, an *erect* image will be formed; if it slope upwards *towards* the eye, an inverted one. If it do not slope either to or from the eye, *i.e.* if it be vertical, each point of the object will appear to be drawn out in a vertical direction, and we have what sailors call "looming."

Inverted images, seen in this way, are magnified as regards height; direct images usually much diminished. But the investigation of the size of the images requires more formidable mathematics than we can introduce here.[1] The reader, however, who has followed us so far, will have no difficulty in understanding how the appearances in fig. 34 below, which are taken from various parts of the works of Scoresby, can be fully explained by one intermediate stratum (or more) between the cold air over a frozen sea, and a warm stratum 50 or 100 feet aloft.

The ordinary mirage of the desert which, from the

[1] See Tait on "Mirage," *Trans. R. S. E.*, 1881, where references to the chief authorities will be found.

apparent reflection of objects, gives the traveller the impression of the existence of a sheet of water, is due to the rarefaction of the air in the immediate neighbourhood of the hot sand. Its theoretical explanation will be made obvious by looking at fig. 33 upside down.

The general explanation of these phenomena we owe mainly to Wollaston, who reproduced the simpler of them by looking along the surface of a brick wall exposed to sunshine; and the more complex by looking through

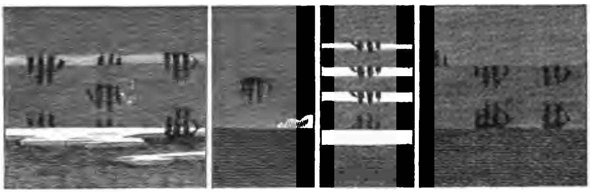

FIG. 34.

the stratum of air close to a long bar of iron which was raised to a high temperature.

179. As another example we take a curious and very instructive problem due to Clerk-Maxwell. Suppose a medium, whose refractive index depends only on the distance from a particular point (and in which, therefore, the surfaces of equal speed are spheres with their centres at that point) to be such that the path of any one ray is a circle; show that the path of every other ray is a circle, and that all rays issuing from any one point come to a focus in another definite point.

By (1) of § 175 we see that all rays lie in planes passing through the common centre of the spheres of equal speed.

To apply (2), remember that *one* ray is circular. For it ρ is a constant (the radius of the circle). Let r be the distance of the corpuscle from the common centre of the spheres, r' the rest of the chord of the path passing through that centre, θ the angle which either end of the chord makes with the corresponding radius of the circular path.

Then, by two well-known properties of the circle,

$$rr' = a^2, \text{ a constant};$$

and
$$r + r' = 2\rho \cos\theta.$$

Also, because μ depends upon r alone,

$$\frac{d\mu}{dn} = \frac{d\mu}{dr} \cos\theta.$$

Eliminate r', θ, and $\frac{d\mu}{dn}$, among these three equations, with the help of the equation in § 175, and we have

$$\frac{2dr}{r + \frac{a^2}{r}} = \frac{d\mu}{\mu}.$$

This gives directly

$$\mu = \frac{a^2 + r^2}{b},$$

where b is an absolute constant.

Hence, *as this is characteristic of the medium*, a must also be an absolute constant.

Thus *all* rays in the medium are circles; and, for every one of them, the rectangle under the segments of a chord passing through the common centre of the spheres of equal speed is the same. Thus all rays leaving any point, at distance r from that common centre, pass through a point at distance a^2/r on the opposite side of the centre.

180. This very singular ideal arrangement was suggested to Clerk-Maxwell by the eye of a fish. He has given an investigation of it, by a totally different analysis, in the *Cambridge and Dublin Mathematical Journal*, vol. ix. As an illustration of those effects of want of homogeneity to which (as already stated) all the complex phenomena of *mirage*, etc., are due, it may be well to consider this simple case more closely. We will therefore consider how images are seen in such a medium. To get rid of the difficulty which would arise from finite change of density if an eye were supposed to be plunged in the medium, we will suppose it to be cut across by a crevasse whose surface is everywhere nearly at right angles to the rays by which the image is to be seen,—the eye being then placed (in air) close to such a cutting surface.

Let AB (fig. 35) be a small object, O the centre of the spherical layers of equal speed, or refractive index. Then every ray from A describes a circle which passes through A′, where AOA′ is a straight line, and

$$AO.OA' = a^2.$$

A similar construction gives B′ from B.

To an eye placed at E_1 (in a little crevasse as before explained), and looking towards the object, it will be seen erect, A being seen in the direction of a tangent to the circle through AE_1A', and similarly for B. Here the rays have not passed through their conjugate focus. But if the eye be now turned away from the object, it (or rather its image) will be seen, A′ in the direction opposite to that in which A was seen, B′ in the opposite direction to B. The image will now be an *inverted* one, but it will easily be seen to possess a strange peculiarity, For what is now seen will be the *back* of the object, the side farthest from the eye. The reader may easily trace for himself the course of the rays which would fall on the eye in any other assigned position. Vision in such cases would usually be of a peculiar character from another point of view, viz., the amount of divergence in the plane of the figure will in general differ from that

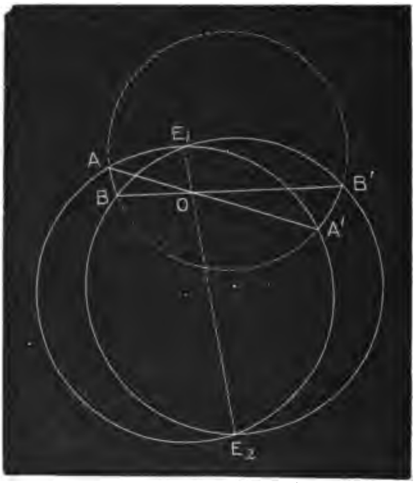

FIG. 35.

perpendicular to its plane, and therefore the rays would have different divergence for the height and for the breadth of the image. These would therefore appear at different distances from the spectator. This, however, could be cured by a proper cylindrical lens. It is clear from this example (which has been chosen for its special simplicity) that want of homogeneity in a refracting medium is capable of producing phenomena of the most extraordinary character.

181. But it was reserved for Sir W. R. Hamilton to discover the existence of what he called the *characteristic function*, by the help of which all optical problems, whether on the corpuscular or on the undulatory theory, are solved by one common process. Hamilton was in possession of the germs of this grand theory some years before 1824, but it was first communicated to the Royal Irish Academy in that year, and published in imperfect instalments some years later. His own description of it,[1] in its relation to the work of his precursors, will be given in the *Appendix*. It is extremely important as showing his views on a very singular part of the more modern history of science.

Without the employment of higher mathematics, it is not possible to show the full merit or utility of Hamilton's discovery. We may merely state that Maupertuis' theorem of *Least* or *Stationary Action* enables us to choose, from the infinite number of paths by which a particle might be caused by frictionless constraint to pass (under given forces) from one assigned point to another, that which will be described if no constraint be applied; and thus compares or contrasts the properties of the natural path with those of forced paths. In *Varying Action* Hamilton deals with all the unconstrained paths which, differing infinitely little either in form or in terminals, can be described under an assigned system of forces. He thus entirely avoids the metaphysical subtleties described in the quotation referred to, and (as regards our present subject) enables us to study a system of rays, or of paths of corpuscles, such as they *are* in nature,

[1] "On a General Method of expressing the Paths of Light and of the Planets." *Dublin University Review*, October 1833.

not such as they *might be made* by imposing external conditions.

182. We have thought it absolutely necessary to point out, even in an elementary work like this, that such a perfectly general method has been developed; but the few fragmentary illustrations of it, which alone can be given without the use of higher mathematics, are so inadequate to the proper exhibition of its power that we do not give them here. We have said enough to show that any one, who wishes really to know the science as it now stands, must previously prepare himself by properly extended mathematical study. When he is possessed of this indispensable instrument, he may boldly attack the precious stores of knowledge already accumulated. There is, as yet, no admission to any but those possessed of this master-key.

CHAPTER XII.

ABSORPTION AND FLUORESCENCE.

183. We must now take up the *third* and *fourth* of the categories under which light incident on the bounding surface of two media may fall :—*scattering* and *absorption*. We take them together, because in the great majority of bodies, as we have already seen, scattering takes place not merely at the surface but within some distance below the surface, which in general is small, but in some cases considerable. And when the scattering takes place, even in part only, below the surface, the scattered light is usually modified by absorption.

184. An excellent instance of this scattering from below the surface is afforded by a mass of thin films or small particles of transparent bodies, such as glass, water, or ice.

Thus pounded glass, froth or foam, snow, clouds, etc., appear brilliantly white in sunlight, and are, in consequence, opaque when in layers of sufficient thickness. Here the light is obviously scattered by reflection. What passes through one film, crystal, or particle is, in part, reflected from the next, and so on.

185. Even when the froth consists of bubbles of a highly-coloured liquid, such as porter, for instance, it

usually shows but slight traces of colour, for the great majority of the scattered rays have passed through very small thicknesses only of the liquid. In the same way, very finely-pounded blue or red glass (unless it be exceedingly deeply coloured when in mass) appears nearly white.

But when a mass of water is full of air bubbles, as, for instance, is the case in the neighbourhood of a breaker, the light reflected from the surfaces of these bubbles suffers a double absorption by the water before it reaches the eye. This is one of the causes of the exquisite colours of the sea.

Near shore, or in shoal water, another cause sometimes comes into play, viz., fine solid particles suspended in the water. When such particles, whether in air or in water, are exceedingly small, they may produce colours due to their mere minuteness, and not alone to their own colour nor to the absorptive properties of the medium. This, however, is a question of physical optics.

186. In general, even the most highly-coloured opaque or translucent solids, such as painted wood or stained paper, are visible by scattered light whatever portion of the spectrum falls on them.

This is very well seen with highly-coloured paper-hangings, when illuminated by homogeneous light, such as that of a sodium flame (a Bunsen flame, into which is thrust a platinum wire dipped in strong brine; or, still better, a piece of metallic sodium in an iron spoon). The red, orange, and yellow parts usually appear very bright under such treatment, the blue parts appearing but slightly illuminated. The colour of all is, of course, that of the incident light. It appears, therefore, that *some* of the light is scattered from the surface. It is by this, for instance, that the blue parts are feebly visible. But that which is scattered from the portions coloured red, orange, etc., must come mainly from under the surface.

187. An excellent proof of this is furnished by mixing, in proper proportions, a yellow and a blue powder, or yellow and blue paints. It is commonly imagined that the green colour which is thus produced is a *mixture* of blue and yellow. Far from it!

When a disk, divided into sectors, alternately coloured with the same blue and yellow pigments, is made to rotate rapidly in its own plane, it, of course, produces on the eye the true result of a mixture of these blue and yellow colours. This depends for its exact tint on the pigments employed, and on the angles of the sectors, but is usually a faint pink or a muddy purple,—utterly different from the green produced by mixing the powders or the paints.

Helmholtz was the first to point out the true source of the green. It is the *one colour* which is not freely absorbed either by the yellow or by the blue pigment. For the scattered light by which the mixture is seen comes chiefly from below the surface, and has thus suffered absorption by each of the component powders. The yellow powder removes the greater part of the blue, indigo, and violet rays (§ 130); the blue, the greater part of the reds, oranges, and yellows. Thus the light which finally escapes is mainly green.

188. For the accurate study of the absorptive power of a solid or liquid medium, it is necessary to compare

the spectrum of white light, which has passed through a plate or layer of it, with a normal spectrum. This is easily effected by placing the absorbing medium (if a fluid, it must be in a glass trough with parallel sides) in front of the narrow slit through which the light passes, and in such a position that one-half of the slit only is thus covered. We have then side by side, under precisely similar circumstances, two spectra to be compared (one altered by absorption, the other not); and very minute differences between them can thus be detected.

When the medium produces a general weakening of the whole spectrum, as well as particular local absorptions, the white light passing through the other half of the slit may be weakened to any desired extent by reflection at the proper incidence from a plate of glass, before it falls on the slit.

189. To give a satisfactory representation of the phenomena of absorption spectra by the help of a woodcut is not easy. The highest artistic skill could not adequately represent the ordinary solar spectrum by the use of the finest pigments. All optical *colour* phenomena must be *seen:* they cannot be reproduced by painting. In such circumstances the simplest method of indicating the locality and amount of the absorption is the best.

As we have already seen that we cannot by the eye judge of the relative intensities of lights which differ much in colour, we shall represent the normal spectrum (for our present purpose) as equally bright throughout, and indicate the absorption at different parts by shading of various degrees of depth. A few of the Fraunhofer lines are introduced to indicate (in the absence of colour) the parts of the spectrum which are attacked by the various absorbents. These lines are, of course, in the same absolute positions in all the various spectra; for the spectra are all supposed to be produced by the same prism. The line B is in the red, D in the orange, E and F in the green, and G in the indigo. They correspond, as we have already said, to perfectly definite kinds of homogeneous light, and therefore adequately represent the distribution of colours in the spectrum, however

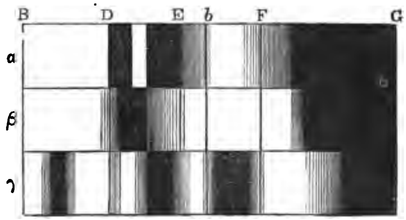

Fig. 36.

much irrationality of dispersion may be shown by the material of the prism.

190. In fig. 36 *a* represents the spectrum of light which has passed through diluted blood; *β* shows the spectrum when the blood has been acted on by a reducing agent; and *γ* the spectrum when the blood has been altered by acidulation with acetic or tartaric acid. These figures are taken from an important paper by Stokes (*Proceedings of the Royal Society*, 1864).

Fig. 37 shows in a rude way the absorption by cobalt glass cut in wedge form, and corrected by an equal prism of clear glass.

191. The commonly received method of calculating the absorption by layers of gradually increasing thick-

ness is to suppose that, if a layer of unit thickness weakens in any ratio the intensity of any particular homogeneous ray, another unit layer will further weaken in the same ratio that which reaches it, and so on. Thus the amount which passes through a number of layers diminishes in geometrical progression, while the number of layers increases in arithmetical progression. This is certainly true (neglecting the amount reflected),

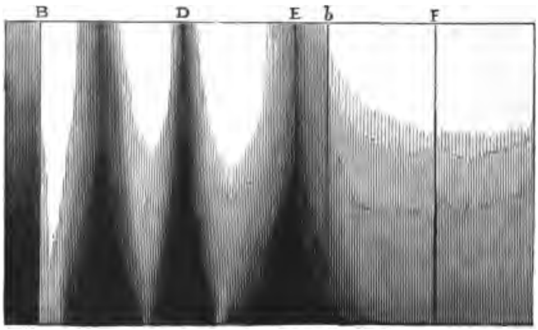

FIG. 37.

unless the *intensity* of the light have an effect on the percentage transmitted.

And fig. 37 shows, in a very striking manner, the difference between similar terms of different geometric series as the common ratio becomes less and less. This ratio is not much less than 1 for certain red and blue rays, is smaller for yellow, and is very small for the rest of the red, for orange, and for green. The latter colours are therefore rapidly got rid of with increasing thickness; then the yellow becomes too feeble to be seen; while, even after the blue becomes almost insensible, the specially favoured red rays are still transmitted in sufficient quantity to be observed.

192. If r be the fraction of any species of homogeneous light which is transmitted by a plate of any substance, of unit thickness, that transmitted by a plate of thickness x (of the same substance) is r^x. The following little table will greatly assist the reader in understanding the relative rapidity of extinction of different rays passing through various thicknesses of an absorbing medium. It is a table of double entry, the first column giving various values of x, and the upper row various values of r, while the corresponding value of r^x is in the same column with that of r and in the same row with that of x.

1	1	0·99	0·9	0·5	0·1
2	1	0·98	0·81	0·25	0·01
5	1	0·951	0·59	0·03	0·00001
10	1	0·904	0·349	0·0009	...
100	1	0·366	0·00003	...	...

Thus a ray, which loses 1 per cent in unit thickness, still preserves more than 90 per cent after passing through ten units. But a ray which loses 10 per cent in the first unit (and which, therefore, will thus far appear scarcely more weakened than the first) is reduced to 35 per cent by passage through ten units. After passing through a hundred units the first ray has lost only 63 per cent, the second is practically invisible.

193. The *assumption* made in § 191 above deserves a word or two of comment, not only because it is essential to the calculations there made (the results of which are, at least approximately, verified by observation), but because it is one of the most important bases of the general theory of radiation. But we defer such comment

to a later chapter, thinking it sufficient for the present to point out that the step is really an assumption. Stokes has suggested an exceedingly simple process for testing its accuracy experimentally, but we are not aware that it has yet been put in practice. This process is based on the assumption (about whose truth there can be no doubt) that the percentage of ordinary light *reflected* under given circumstances is independent of the intensity of the light. If, then, the light be both reflected and subjected to absorption, and if both assumptions above are correct, the final intensity ought to be independent of the *order* in which these operations are applied. Let a beam of sunlight be reflected from a plate of glass blackened at the back, at an incidence as nearly direct as possible, so as (§ 74) to be very much weakened (to about $\frac{1}{25}$th in fact). Cut a uniform plate of the absorbing medium into two parts, one of which is to be interposed in the way of half the beam *before* it is reflected; the other in the path of the remaining half *after* it is reflected; and the eye will enable us to judge, with very great accuracy, as to the relative intensity of the two parts of the beam. This follows from the facts stated in § 62, because in this case the colour is necessarily exactly the same in the two beams compared.

194. In thin plates cobalt glass is blue, because the particular red, which it does not absorb freely, forms only a small fraction of the whole transmitted rays; while in thick masses it is nearly red, for then little but this favoured red is transmitted. For a similar reason Condy's fluid (permanganate of potash) changes its tint in a very singular manner (even when preserved from the action of the air) by gradual dilution with water.

The imperfection of the achromatism of the eye is readily proved by looking through a plate of cobalt glass at a small hole in the window-shutter of a dark room. The hole at first appears red with a blue space round it; but, by an effort of the muscles of the eye, we can see the hole blue, and then there is a red space surrounding it. Rays of so widely different refractive index cannot be seen in focus simultaneously.

Very curious effects are produced when we examine a landscape through such a glass. Foliage of certain kinds scatters scarcely any blue rays, and therefore appears reddish. Bluish-greens, again, which scatter very little red, appear blue. The effects may be exaggerated in a very striking degree by combining the absorptions of two or more media, so as to allow of the free transmission of a few far-detached portions of the spectrum.

195. Brewster made the singular discovery that a solution of oxalate of chromium and potash (§ 196) produces one solitary narrow absorption-band, almost resembling one of the broader lines in the solar spectrum. Certain solutions of salts of didymium, etc., possess a similar property. These may be used in the absence of sunlight, as Brewster suggested, for the purpose of measuring with great exactness refractive indices, etc., by means of artificial light. But the fundamental principle of spectrum analysis, as will be seen later, furnishes us with a still more effective process.

196. Closely connected with intense local absorption in certain parts of the spectrum, is the phenomenon of *Abnormal Dispersion*, one of the most singular discoveries of modern times. It seems to have been first observed

by Fox Talbot; and he discovered its real nature. But the first published notice of such phenomena is due to Le Roux. Christiansen and others have since greatly extended our knowledge of the subject, and Helmholtz and Ketteler have given theoretical explanations of it. Fox Talbot's experiment, though the earliest on record, is one of the easiest to perform, and we therefore quote his own account. The experiment was made about 1840, and the following account is from the *Proc. Roy. Soc. Edin.*, 1870-71.

"I prepared some square pieces of window glass, about an inch square. Taking one of these, I placed upon it a drop of a strong solution of some salt of chromium, which, if I remember rightly, was the double oxalate of chromium and potash, but it may have been that substance more or less modified. By placing a second square of glass on the first, the drop was spread out in a thin film, but it was prevented from becoming too thin by four pellets of wax placed at the corners of the square, which likewise served to hold the two pieces of glass together. The glasses were then laid aside for some hours until crystals formed in the liquid. These were necessarily thin, since their thickness was limited by the interval between the glasses. Of course the central part of each crystal, except the smallest ones, was bounded by parallel planes, but the extremities were bevilled at various angles, forming so many little prisms, the smallest of them floating in the liquid. When a distant candle was viewed through these glasses, having the little prisms interposed, a great number of spectra became visible, caused by the inclined edges. Most of these were no doubt very imperfect, but by trying the glass at various points, some very distinct spectra were met with, and these could with some trouble be isolated by covering the glass with a card pierced with a pin-hole. It was then seen that each prism (or oblique edge of crystal) produced two spectra oppositely polarised and widely separated. One of these spectra was normal; there was nothing particular about it. The colours of the other were very anomalous, and, after many experiments, I came to the conclusion that they could only be explained by the supposition that the spectrum, after proceeding for a certain distance, stopped short and returned upon itself."

197. Le Roux in 1860[1] discovered that vapour of iodine, which allows only red and blue rays to pass, refracts the red more than the blue. He, like Talbot, did not at first venture to publish his result, and it appeared only in 1862. Among the many convincing proofs of its accuracy, he shows that the dispersion by an iodine-vapour prism can be nearly achromatised by a glass prism which gives refraction in the *same* direction. He also states that the dispersion in iodine-vapour is less as the temperature is higher.

Christiansen's[2] earliest determinations were made in 1870 upon an alcoholic solution of fuchsine (one of the powerful aniline colours). This solution gives a dark absorption-band in the green; and it was found that the refractive index rises (as in normal bodies) for rays from the red to the yellow. But all the rest of the transmitted light, consisting of the so-called "more refrangible" rays, is less refracted than the red. Kundt and others shortly afterwards greatly extended these observations.

[1] *Comptes Rendus*, lv., 1862. [2] *Pogg. Ann.*, cxli.

198. The explanation of this phenomenon, which has been advanced by Helmholtz,[1] depends upon an assumption as to the nature of the mutual action between the luminiferous ether and the particles of the absorbing medium, coupled with a further assumption connecting the absorption itself with a species of friction among the parts of each absorbing particle.

In 1879 De Klerker[2] made a very curious observation, which shows that the whole subject is still to some extent obscure. He employed two hollow prisms of equal angle, turned opposite ways, and filled with alcohol. Through such a combination light passes (as we have seen) without refraction or dispersion. When a few drops of the fuchsine solution were added to the contents of one of the prisms, the yellow, orange, and red rays (in the order named) began to separate themselves from the others. This process could be carried on until the solution was so strong that it transmitted no visible light. All this time the blue and violet rays remained apparently unrefracted—the yellow, orange, and red showing continually increasing refraction. The conclusion from this, on either theory of light, is that the addition of fuchsine to alcohol alters the speed of propagation of the (so-called) less refrangible rays, but not perceptibly that of the more refrangible.

199. The singular surface-appearances presented by "canary" glass, by some specimens of fluor-spar, and by certain liquids, such as a solution of sulphate of quinine acidulated with sulphuric acid, had been the source of much speculation by Brewster, Herschel, and others, long before their true nature was traced by

[1] *Pogg. Ann.*, clv., 1874. [2] *Comptes Rendus*, 1879.

Stokes in 1852.[1] By a series of well-contrived experiments, one or two of which will presently be described, he put it beyond doubt that the cause of these phenomena lies in a change of refrangibility of the light which has been absorbed by the upper layers of the medium, and then given off again. In every case the fluorescent light appears to belong to a less refrangible part of the spectrum than does the incident light which gave rise to it, thus affording an instance of dissipation, or degradation of energy. When a very powerful beam of light passes through a fluorescent body, its whole track may thus be distinguished, though, unless it be convergent, the illumination becomes rapidly feebler as the beam penetrates farther. The fluorescent light, thus seen in the interior of the body, must be carefully distinguished from the light merely scattered by impurities, as by dust in the air or by fine particles suspended in water. Here there is no change of wave-length, as in true fluorescence, though there are remarkable effects of another kind which will be examined later.

200. The yellowish-green surface-colour of canary glass (coloured with oxide of uranium) is well known, as the substance is, mainly on account of this property, very commonly used for ornaments. If we admit a ray of sunlight (or light from the electric lamp) into a dark room through a cobalt glass, so dark that the feeble violet-coloured light it transmits is scarcely visible, we find that the canary glass shows its yellow-green colour vividly when placed in the track of the ray. Striking as this experiment is, it is not quite conclusive as to the true cause of the appearance.

[1] *Phil. Trans.*, "On the Change of Refrangibility of Light."

But if we take another piece of glass, slightly tinged of a brownish-yellow (by oxide of gold), we find that it is quite transparent to the brilliant light from the canary glass; if, however, we place it in the track of the violet rays *before* they fall on the uranium glass, it prevents the production of the phenomenon altogether. That is, rays which cannot pass through the glass coloured with gold are rendered capable of freely passing through it after incidence on the canary glass. That the phenomenon is due to rays which are stopped by the uranium glass itself, is proved by the fact that a second piece of the glass, placed in the track of the rays which have passed through the first, does not show the phenomenon. Unless, indeed, the source of light be very bright, or the beam highly concentrated, the appearance is confined to a mere surface-layer of the first piece of canary glass. The phenomenon is very well shown by an aqueous infusion of horse-chestnut bark. Some specimens of paraffin oil exhibit it most brilliantly.

201. To find the rays which are most effective in producing the fluorescence of any substance, we have only to place it in a pure spectrum of sunlight (or, preferably, of the electric light),—prisms and lenses of quartz being used for producing the spectrum, because that material is found to be far less opaque than glass is to the violet and ultra-violet rays. When this is done with uranium glass we find scarcely a trace of effect until the substance reaches the blue rays, and the effect persists through all the higher colours, and even very considerably beyond the bounds of the visible spectrum. Stokes in fact used it as a means of studying

the otherwise invisible, but far extending, spectrum of the ultra-violet rays of the electric spark.

The mechanism of the process by which these extraordinary results are produced is still somewhat obscure. Stokes has, however, shown that, if a vibrating system, which is incapable of propagating waves of short period, be acted on by such waves, there occurs a sort of compromise, in which the parts of the system acted on are thrown into a species of *congested* oscillation, whose period is, in all cases, longer than that of the exciting cause.

With Professor Stokes' permission we print the following extract from a recent letter of his:—

"I have long believed that the explanation of fluorescence lay in the communication of motion by means of the intermolecular forces, from the molecule or part of a molecule thrown into agitation by the vibrating ether, to the neighbouring parts, and thence to the mass in general. A great many years ago, to test this idea in some simple dynamical system, I worked out the following problem, which may be solved without difficulty:—

"Imagine an infinite string, without weight, stretched by a uniform tension and loaded at equal intervals by an infinite number of equal masses regarded as points. Let one of the masses be acted on continually by a small transverse disturbing force, expressed by the sine of an angle proportional to the time, and let it be required to determine the corresponding small periodic motion of the system.

"The result is very remarkable in relation to the physical question. Suppose, in the first instance, that we consider the possible simple harmonic motions of the system when there is no disturbing force, taking for simplicity the motion as in one plane. We have of course a transcendental equation, with an infinite number of real roots, to determine the periodic time; the smallest, say T, corresponding to the case in which the masses move alternately in opposite directions through equal spaces. Now the motion, in the case of the disturbing force, is of quite a different character according as the periodic time is greater or less than T. In the former case the disturbance extends infinitely in both directions; we have, in fact, a sort of undulation propagated both ways from the disturbed mass. In the latter the disturbance is local, decreasing indefinitely as we recede from the mass on which the force acts.

"It is needless to say that the solution may be extended by Fourier's theorem to the case of a disturbing force which is an arbitrary function of the time. But without that we can see the general effect of a periodic disturbing force acting for a great number of periods on one mass, and then ceasing.

"If the period be greater than T, any nascent disturbance is carried off to a distance by undulations, and the disturbance remains insensible; since I suppose that the disturbance which would be produced by the force, acting for a single period, is insensibly small, and that it is only by the continued action of the disturbing force that the disturbance of the masses can become sensible. I make this supposition because we have every reason to believe that this is what is actually true of the disturbance of ponderable molecules excited by ethereal undulations.

"But if the period of the disturbing force be less than

T, though a small amount of disturbance of the masses may at first be propagated off, the disturbance will gradually assume its permanent form as a local agitation. If the disturbing force now cease to act we shall be left, as regards the subsequent motion of the masses, with an initial disturbance of a local character. The subsequent motion will be expressed by an infinite series of simple harmonic terms as regards the time, the smallest period being T.

"This strikingly illustrates the law that the refrangibility of the light due to fluorescence is always less than that of the exciting light. Moreover, in dealing with a single fluorescent substance—not a mixture of two or more—I have generally found that the following feature is (very approximately, at any rate) observed:—As we take incident light of increasing refrangibility it is at first inactive; then, on reaching a certain point, P, of the spectrum, it begins to produce fluorescence, and the heterogeneous fluorescent light contains refrangibilities not extending beyond P. As we continue to progress in the incident spectrum the highest refrangibility of the fluorescent light does not follow the refrangibility of the incident light, but remains about P."

202. The duration of fluorescence is so very short that it is only by specially-devised methods that we can make certain that it persists for any measurable time after the exciting light is cut off from the fluorescent body.

Becquerel's ingenious *Phosphoroscope* was invented for the purpose of inquiries of this kind. It consists essentially of a shallow drum, in whose ends two eccentric holes, exactly opposite to one another, are cut. Inside it are fixed two equal metal disks, attached perpendicularly to an axis, and divided into the same number of sectors, the alternate sectors of each being cut out. One of these disks is close to one end of the drum, the other to the opposite end, and the sectors are so arranged that, when the disks are made to rotate, the hole in one end is open while that in the other is closed, and *vice versâ*. If the eye be placed near one hole, and a ray of sunlight be admitted by the other, it is obvious that while the sun shines on an object inside the drum the aperture next the eye is closed, and *vice versâ*. If the disks be made to revolve with great velocity by means of a train of toothed wheels, the object will be presented to the eye almost instantly after it has been exposed to sunlight; and these presentations succeed one another so rapidly as to produce a sense of continued vision.

By means of this apparatus we can test with considerable accuracy the duration of the phenomenon after the light has been cut off. For such a purpose we require merely to know the number of sectors in the disks and the rate at which they are turned. To guard against deception by the persistence of impressions on the retina, the eye should not be directed fixedly on the object, but should be kept travelling slowly round the position in which it is seen to lie.

203. Uranium glass shows, with rapid turning, nearly as vivid an effect as when exposed to continuous light, but fades fast when the speed of the rotation falls off. A pinkish kind of ruby, exposed to concentrated sunlight in the phosphoroscope, is seen to glow with a bright red like a piece of live coal.

With very rapid turning, feeble fluorescence can be detected in a great many substances in which the ordinary methods will not show it. This is due in great measure to the fact that the phosphoroscope entirely does away with the scattered light which, in the ordinary mode of examining these substances, overpowers their feeble fluorescence.

204. What is correctly termed phosphorescence has nothing to do with phosphorus (whose luminosity in the dark is due to slow oxidation), but it is merely a species of fluorescence which lasts for a much longer time after the excitation has ceased than does that just described.

Pliny speaks of various gems which shine with a light of their own, and Albertus Magnus knew that the diamond becomes phosphorescent when moderately heated. But the first discovery of phosphorescent substances, such as are now so common, belongs to the early part of the seventeenth century. During that century the Bologna stone (sulphide of barium) and Homberg's phosphorus (chloride of calcium) were discovered. Canton's phosphorus (sulphide of calcium) dates from 1768. To the substances mentioned may now be added sulphide of strontium.

Any of these sulphides, which must be carefully preserved from the air in sealed glass tubes, appears brilliantly luminous when carried from sunlight into a dark room, and for a long time after presents the general aspect of a hot body cooling. The rays which excite their luminosity are (as with the generality of fluorescent bodies) those of higher refrangibilities; but the colours of the phosphorescent light are of the most varied kind, even in specimens of almost precisely the same chemical composition, but prepared at different times.

The causes of this strange diversity are as yet quite unguessed at; but the property has been taken advantage of for the production of what are called *luminous paints.* The behaviour of these substances is one of the most singular phenomena in optics. How they manage to store up so large a supply of energy during a short exposure to bright light, and to dole it out continuously for so long a time and mainly in the form of light, is exceedingly puzzling, especially as no other physical or chemical change has yet been found to accompany the process.

Another curious fact connected with their behaviour was discovered by Becquerel. He found that the less refrangible rays have in some cases the power of arresting the emission of light from these bodies when they have been previously excited by higher rays.

CHAPTER XIII.

PRELIMINARY REMARKS ON THE UNDULATORY THEORY.

205. THE explanation of the fundamental laws of Geometrical Optics by the wave-theory requires some preliminary remarks. We confine ourselves to what is strictly necessary for the immediate purposes of the present work. Nothing beyond an approximation will be attempted, where further investigation requires higher mathematics.

The great difficulty which meets us at the very outset, and which sufficed to make Newton reject the undulatory theory, is the fact that in general light moves in straight lines in air, and does not spread in all directions like other wave-disturbances, such as sound and water-waves, after passing through an aperture. The *full* treatment of this question requires an amount of mathematics which we cannot introduce here; but the general nature of the explanation (which depends upon the smallness of the wave-lengths of light even as compared with the diameter of a pin-hole) will be obvious from some of the approximate investigations which follow. For the present we will simply admit the existence of the difficulty, and proceed with the explanation of other phenomena. The reader, who has in this way gained some confidence in the theory from its agreement with facts of the most diverse kinds, will be led back to the consideration of this preliminary difficulty. In the next article we will state the more immediately important bases of the undulatory theory.

206. (*a*) The essential characteristic of wave-motion is that a disturbance of some kind is handed on from one portion of a solid or fluid mass to another. In certain cases only, this disturbance is unaltered in amount and in character as it proceeds.

(*b*) So far as light is concerned, the speed with which each particular species of disturbance passes, in any direction, through a homogeneous isotropic medium is constant, and is the same for all directions. When the medium is not homogeneous, the speed may vary from point to point. If the medium be not isotropic, the speed may depend upon the *direction* of propagation. In interplanetary spaces, where there is probably no ordinary matter, except in the form of sporadic groups of meteoric masses, large and small, the speed seems to be independent of the particular species of wave. Examples of each of these peculiarities will be met with presently.

(*c*) When two or more separate disturbances simultaneously affect the same portion of a medium, the effect may be very complex. But, in the case of light, it has been found that a geometrical (or rather *kinematical*) superposition or composition agrees, at least to the degree of accuracy of the experiments, with all the observed facts. This would be the case, as a dynamical result, if the distortions due to wave-motion were always, even for the most powerful light, exceedingly small. On this is based the whole doctrine of *Interference*, Young's grandest contribution to the wave-theory (1801). It follows from this that any number of separate disturbances may be propagated *through* one another in the same portion of the luminiferous medium; each emerging from that portion as if it had not encountered the others. [But this is not necessarily true, as will be seen later, if the portion spoken of be *maintained* in certain definite forms of disturbance.]

(*d*) The disturbance at any point of a medium, at any instant, is that due to the superposition of all the disturbances which reached it at that instant from the various surrounding parts of the medium. This is (in a somewhat generalised form) what is commonly known as Huygens' principle, first enunciated in 1678.

(*e*) The *front* of a wave is defined, at any instant, as the continuous locus of all portions of the medium which, at that instant, are equally and similarly distorted, and have equal velocities.

The word *continuous* is inserted because, in oscillatory wave-motion, such as that of light, a large number of successive waves are exactly equal and similar to one another. Thus we have a *series* of wave-fronts following one another, which are not to be considered as parts of one wave-front. The distance between two successive fronts in which the distortions, and also the velocities, are equal, measured in the direction in which the light is travelling, is called the *wave-length*. Henceforth we restrict ourselves to oscillatory waves, unless the contrary be specified.

(*f*) The *colour* of homogeneous light depends entirely on the *period* of a wave, *i.e.* on the time of passage from one wave-front to the next. This is obviously the same thing as the time of a complete vibration of any one particle of the medium:—whatever be the speed of light in the medium, or the consequent wave-length.

207. These being premised, let us consider the propagation of homogeneous light from a luminous point in a homogeneous isotropic medium. Here we have simply a succession of concentric spherical wave-fronts, their radii differing by one or more whole wave-lengths. In accordance with what has been said above, we might assume that the disturbance in any portion of one of these fronts is propagated radially. But we may consider it from a different point of view, as hinted in (*d*) above. Simple as this particular case is, the reader will probably find that it will greatly assist him in understanding the more complex ones which follow.

208. Every disturbed portion of the medium may be looked upon as a centre of disturbance from which a new set of spherical waves is constantly spreading. Take then, as common radius, the space described by a disturbance in any very short interval; and, with centres at every point of any one wave-front, describe a series of spheres. The ultimate intersections of these spheres will lie on a surface which is the *envelop* of them all. In the case considered, it is obviously a sphere whose radius exceeds that of the wave-front from which we started by the common radius of the set of spheres. This is shown in a central section in fig. 38 below, which suffices to prove that we arrive (*by this mode of construction*) at the result which we know in this simple case to be the correct one. It will be seen that the centres of the construction-spheres lie on a certain part of one wave-front, while their ultimate intersections lie

on the *corresponding* part of the future wave-front. This holds for spheres of all radii, and for continually increasing radii shows that a *plane* wave moves perpendicularly to its front. This is so important a part of Huygens' work that we give his own words (*Traité de la Lumière*, 1690, pp. 18-20) in an *Appendix*.

209. We may, however, here give the main features

FIG. 38.

of this extremely important statement in the following much condensed form :—

Each part of the wave-front must so extend itself that its extremities may always lie on lines diverging from the source. Thus the part BG, of the wave-front whose centre is A, must extend itself into CE, which is bounded by the same lines ABC and AGE. For, although special waves extend outwards from the space CAE, they do not conspire to produce a resultant wave except precisely along CE, their common tangent.

And thus we see why light is propagated in straight lines, for the detached wavelets which diverge from the line are too feeble to produce light.

However small the opening BG may be, the same conclusion must hold; because, small as it may be, the opening is large enough to contain a large number of particles of the ether, which are of inconceivable minuteness.

Besides, what has been said about the feebleness of the special wavelets shows that it is not necessary to suppose the particles of the ether to be all of one size, though such a supposition would better suit the propagation of the motion. For though, when small particles impinge on larger ones, there may be recoil, this will produce only detached wavelets moving backwards to the source, too feeble to produce light, and not one wave made up of a number of others, such as is CE.

Another, perhaps the most marvellous, property of light is that when (rays) come from different, even opposite, directions they pass through one another without impediment. Thus a great number of persons may simultaneously see, through the same aperture, each a different object; and two persons can simultaneously see one another's eyes.

210. More recent investigation has shown that some of these statements are by no means exact; but the more prominent of their inaccuracies will be obvious to the reader as he proceeds, so that we need not discuss them here. It is only necessary to say, for the present, that the limitation of light to a *ray* is due to the excessive minuteness of the wave-lengths of every visible radiation. When the aperture through which light passes is of the same order of minuteness as the wave-length, the phenomena undergo a complete change of character.

211. We will now, for the purposes of this elementary work, assume that Huygens' mode of finding one wave-

surface from a preceding one is applicable in all cases, and will not trouble ourselves with the fact that our construction, if fully carried out, would indicate a retrograding wave as well as a progressive one.

The obvious fact that a *solitary* wave can be propagated in water, or along a stretched string, may assist the reader in taking the bold step which we have proposed to him.

And we will also assume that this mode of representation leads to correct results, even when we do not choose a wave-front as the locus of the centres of disturbance:—that in fact we may choose for our purpose *any* surface through which the rays pass, provided always that the radii of the construction-spheres are so chosen that the length of each ray from some definite wave-front to the centre of the sphere, together with the radius of that sphere, always corresponds to a path described in a given time. [See, again, § 206 (*d*).]

212. We are now prepared to explain the reflection of light, and we need do so for a plane reflecting surface alone, with plane waves impinging on it; because the length of a wave, as we shall soon see, is an almost vanishing quantity in comparison with the radius of curvature of any artificial mirror, be it even a very small drop of mercury.

213. Let a plane wave-front be approaching a plane mirror, and at any instant let fig. 39 represent a section by a plane perpendicular to each, cutting the wave-front in AB and the mirror in AC. From what has been already said, the motion of every part of AB is perpendicular to that line, and in the plane of the figure. During the time that the disturbance at B takes to reach

C, the disturbance which had reached A will have (in part, for there is usually a refracted part also) spread back into the medium in the form of a spherical wave whose radius, AD, is equal to BC. Its section is of course a circle. That from any other point P will have reached Q, and then (in part) diverged into a spherical wave whose centre is Q and radius QT (=QT′)=BC−PQ. Obviously all the circles which can be thus drawn ultimately intersect in the straight line CD. This is a sec-

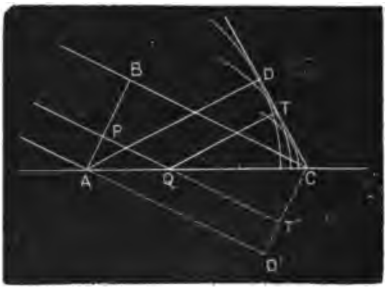

Fig. 39.

tion of the reflected wave-front. A plane wave, therefore, remains a plane wave after reflection; each part of it obviously moves in the plane of incidence; and the similarity of the triangles ABC and CDA proves the equality of the angles of incidence and reflection, for the ray is everywhere perpendicular to the wave-front. It is to be particularly noted that this is independent of the velocity of the light, so that all rays are reflected alike. [In this, as in the preceding and the immediately following instances, the diagram has been taken (with but slight change) from Huygens.]

214. This being true of any *plane* wave-front, large

or small in area, is necessarily also true of any wave-front of finite curvature. Thus, if a set of rays be drawn perpendicular to any wave-front, they will after reflection be perpendicular to a new wave-front; and the lengths of all the rays, from wave-front to wave-front, will be equal. [See, again, § 82.]

This is merely another way of stating that if a set of rays can be cut at right angles by a surface (of continuous curvature) they will always be capable of being cut at

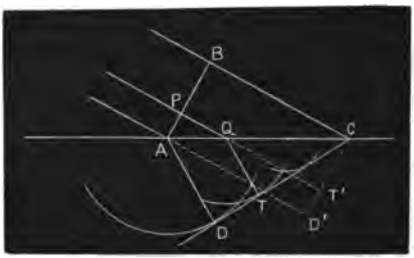

Fig. 40.

right angles by such a surface, even after any number of reflections at surfaces of finite curvature, provided they move in a homogeneous isotropic medium.

This proposition will be seen to be capable of extension to refraction, provided always that both media are homogeneous and isotropic.

215. For a plane wave, falling on a plane refracting surface, our construction (fig. 40) is as follows:—

Let AB be, as before, a plane wave-front in the first medium, and AC the plane surface of the second medium. As before, let BC be perpendicular to AB. Also let CD′ be drawn parallel to BA. With centre A, and radius AD equal to the space described in the second medium while

BC is described in the first, let a sphere be described. The disturbance at A will have diverged in this sphere, while that at B has just reached C. The disturbance at any other point, as P, will have passed to Q, and thence have diverged into a sphere, of radius QT such that

$$QT : QT' :: AD : BC.$$

Obviously all spheres so drawn ultimately intersect along CD, which is therefore the front of the refracted wave. The angles of incidence and refraction, being the inclinations of the incident and refracted rays to the normal, are the inclinations BAC and DCA of the incident and refracted wave-fronts to the refracting surface. Their sines are evidently in the ratio of BC to AD, *i.e.* they are *directly* as the speeds of propagation in the two media.

216. Hence the law of refraction also follows from this hypothesis. But there will now be separation of the various homogeneous rays, because the ratio of their speeds in the two media depends on the wave-length.

Besides, it is clear from the investigation above that, in the refracting medium, the rays are still perpendicular to the wave-front. Thus the proposition lately given may now be extended in the following form:—

If a series of rays of homogeneous light, travelling in homogeneous isotropic media, be at any place normal to a wave-front, they will possess the same property after any number of reflections and refractions. [§ 82.]

And it is clear from the investigations already given that the *time* employed by light in passing from one of these wave-fronts to another is the same for every ray of the series.

217. We now see how crucial a test of theory is furnished by the simple refraction of light. On the corpuscular theory the speed of light in water is to its speed in air as 4 : 3 nearly; on the undulatory theory these speeds are as 3 : 4; since, as we have seen, the refractive index of water is about $\frac{4}{3}$. But Foucault's experimental method (§ 70) showed at once that the speed is less in water than in air. This finally disposed of the corpuscular theory.

Though it had been conclusively disproved long before, by certain interference experiments whose nature will presently be described, the argument from these was somewhat indirect and not well suited to convince the large non-mathematical class among optical students and experimenters.

218. The true author of the undulatory theory is undoubtedly Huygens. Grimaldi, Hooke, and others had expressed more or less obscure notions on the subject, but Huygens (in 1678) first gave it in a definite form, based to a great extent upon measurements of his own. His tract on the subject was read in that year to the French Academy, but not published till 1690, when it appeared with the title *Traité de la Lumière.*

Huygens gives the explanation of the double refraction of Iceland spar, which had been described by Bartholinus in 1670.

Unfortunately, the remarkable step taken by Newton in explaining the law of refraction on the corpuscular theory,—the earliest solution of a problem connected with molecular forces,—had for some time been before the scientific world. The authority of Newton was paramount in such matters, and the work of Huygens produced no effect at the time of its publication.

Even the genius of Young, who, at the commencement of the present century, recalled attention to this all-but-forgotten theory, and enriched it by the addition of the principle of interference, as well as by many important applications, failed to secure its recognition.

219. It was not till 1815 and subsequent years that, in the hands of Fresnel, the undulatory theory finally triumphed; and, even then, the battle was won against determined resistance on the part of the upholders of the corpuscular theory. Witness what Laplace[1] said, in 1817, in a letter to Young. We paraphrase part of the excerpt, but it is given in the original in the *Appendix*.

"However ingenious the reasoning by which you try to show that, on the wave-theory, the sines of the angles of incidence and refraction are in a constant ratio, I can look on it as an illustration only, and not as a demonstration. I maintain that the problem of the propagation of waves from one medium to another has not yet been solved, and that it is perhaps beyond the present powers of analysis. Descartes explained this constant ratio by the help of two hypotheses; but, as he did not base either of them on dynamical laws, his explanation was strongly opposed and rejected by the majority of physicists, until Newton showed that the hypotheses resulted from the action of the refracting medium on light.

[1] Young's *Works*, ed. by Peacock, vol. i. p. 374. It is matter for curious remark that Laplace refers to Descartes only, and not to Huygens.

"*Then* we obtained a mathematical explanation of the phenomenon on the corpuscular theory:—a theory which, besides, gives the most simple explanation of aberration, a phenomenon not explained by the wave-theory.

"Thus the hypotheses of Descartes, like many of Kepler's guesses about the solar system, have been verified by analysis; but the credit of discovery belongs entirely to him who demonstrates.

"I allow that there are novel phenomena of light still very hard to explain; but careful study will perhaps some day discover the properties of the luminous corpuscles on which their mathematical demonstration depends.

"To go back from phenomena to laws, and from laws to forces, is, as you know, the true course of physical science."

220. Poggendorff remarks that there is no other instance, in the whole history of modern physics, in which the truth was so long kept down by authority.

Poggendorff further remarks that, of the six chief phenomena of light known in Huygens' time, he fully explained three:—reflection, refraction, and the double refraction of Iceland spar—at least so far as concerns the *direction* of the reflected or refracted rays.

Phenomena such as diffraction, and the colours of thin plates, required for their explanation the principle of interference, which was first given by Young; and dispersion (not yet quite satisfactorily disposed of) was first, in a way, accounted for in comparatively recent times by Cauchy.

Huygens himself was the discoverer of polarisation, but he could not account for it. Even Young also, because (like Huygens) he supposed the displacements to be in the direction of the ray, failed to account for it; and it was not explained till Fresnel reintroduced, with the most brilliant success, a guess of Hooke's (of date 1672), that the vibrations of light in an isotropic medium are *perpendicular* to the direction of the ray.

CHAPTER XIV.

INTERFERENCE.

221. TAKING the undulatory theory as the only one left possible by the experiments of Foucault, we will now consider the explanation it offers of various phenomena.

It will be remembered that *we have as yet made no assumption whatever as to the precise nature of a wave;* and it will be found that a large class of important phenomena can be explained by the wave-theory without our making any such assumption; but that other classes of phenomena compel us to adopt certain limitations of the very general hypothesis with which we started.

222. As long as we deal with the first class of phenomena, we may take for granted those properties which are common to all ordinary forms of wave-motion, such as those in water or in air.

In ordinary water-waves the motion of a particle is partly to and fro in the direction in which a wave is travelling, partly up and down, and therefore perpendicular to that direction. This is obvious to every one who watches a floating cork.

In sound-waves, whether in air or in water, the displacement of each particle of the medium is wholly in the direction in which the wave is travelling.

Directly connected with this there is another distinction between these classes of waves. In ordinary water-waves the water-elements change only their *form* as the wave passes; in sound-waves there is change of *volume* also.

A third distinction, also directly connected with the first, is that sound-waves in water travel at a much greater rate than the swiftest, *i.e.* the longest, of oscillatory surface-waves.

223. But, in either case, when two series of waves arrive at a common point, they *interfere*, as it is called, with one another; so that the actual disturbance of the medium at any instant is the resultant of the disturbances which it would have suffered at that instant from the two series separately.

Thus if the two series be equal and similar, and if crests, and therefore troughs, arrive simultaneously from them, the result is a doubled amount of disturbance. If, on the contrary, a crest of the first series arrive along with a trough of the second, the next trough of the first series will arrive along with the next crest of the second, and so on. One series is then said to be half a wave-length behind the other. In this case the portion of the medium considered will remain undisturbed.

224. Thus, at the port of Batsha in Tong-king, the ocean tide-wave arrives by two different channels, one part being nearly six hours, or half a wave-length, behind the other. As a result, there is scarcely any noticeable tide at Batsha itself, though at places not very far from it the rise and fall are considerable. This was known to Newton, and is noticed by him in the *Principia*, iii. 24. See also *Phil. Trans.*, vol. xiv. p. 677, for the observed

facts and Halley's comments. Thus, also, two sounds of the same wave-length and of equal intensity produce silence, if they reach the external ear with an interval of half a wave-length, or any odd multiple of half a wave-length.

225. It is not remarkable that Young's Bakerian Lecture (1801), in which the principle of interference is for the first time described and applied, should consist in great part of extracts from the *Principia*. For there are many passages in Newton's works which might have been written by an upholder of the wave-theory. Unaccountably, however, Newton in the context almost always brings in a reference to the "rays of light" as something different from the vibrations of the ether, yet capable of being acted on by them so as to be put into "fits of easy reflection or of easy transmission." These allusions are the most obscure parts of all Newton's scientific writings; and it is very difficult to form a precise conception of what he meant to express in them.

226. The following passage, extracted from Young's temperate reply (*Works*, vol. i. p. 202) to the violent but ignorant assault made on him by Lord Brougham in the *Edinburgh Review*, is chosen as showing his own estimate of his own work and of its relation to what was already known:—

"It was in May 1801 that I discovered, by reflecting on the beautiful experiments of Newton, a law which appears to me to account for a greater variety of interesting phenomena than any other optical principle that has yet been made known. I shall endeavour to explain this law by a comparison.

"Suppose a number of equal waves of water to move upon the surface of a stagnant lake, with a certain constant velocity, and to enter a narrow channel leading out of the lake. Suppose then another similar cause to have excited another equal series of waves, which arrive at the same channel, with the same velocity, and at the same time with the first. Neither series of waves will destroy the other, but their effects will be combined: if they enter the channel in such a manner that the elevations of one series coincide with those of the other, they must together produce a series of greater joint elevations; but if the elevations of one series are so situated as to correspond to the depressions of the other, they must exactly fill up those depressions, and the surface of the water must remain smooth: at least I can discover no alternative, either from theory or from experiment.

"Now I maintain that similar effects take place whenever two portions of light are thus mixed; and this I call the general law of the interference of light. I have shown that this law agrees, most accurately, with the measures recorded in Newton's Optics, relative to the colours of transparent substances, observed under circumstances which had never before been subjected to calculation, and with a great diversity of other experiments never before explained. This, I assert, is a most powerful argument in favour of the theory which I had before revived: there was nothing that could have led to it in any author with whom I am acquainted, except some imperfect hints in those inexhaustible but neglected mines of nascent inventions, the works of the great Dr. Robert Hooke, which had never occurred to me at the time that I discovered the law; and except the Newtonian explanation of the combinations of tides in the port of Batsha."

227. We are, therefore, called upon to regard light as propagated by some species of regular wave-motion, though we have as yet no hint as to the particular species. For the present, we shall not be concerned with the exact nature of the waves; the period, and the wave-length (which are deducible one from the other, if we know the speed of light in the medium considered) are to be the main objects of our inquiries. And the result to which we are led is very remarkable indeed.

228. Young's first application of the principle of interference was made to the colours of *striated surfaces*, the next to the colours of *thin plates*, and to the *Diffraction Fringes* (first studied by Grimaldi, and afterwards by Newton) which are seen near what would be, if light moved rigorously in straight lines, the boundary of the geometric shadow, when the source is a mere point. Young's explanation of this last phenomenon was greatly improved by Fresnel, but it is too complex for a work like this; so we confine ourselves to the two first. These, however, are not so easily intelligible as the application to an experiment devised by Fresnel several years later.

We therefore commence with Fresnel's experiment, which gives the most simple arrangement yet contrived, but it must be understood that the explanation is really due to Young.

229. BCD (fig. 41) is an isosceles prism of glass, with the angle at C very little less than two right angles. A luminous point is placed at O, in the plane through the obtuse edge of the prism and perpendicular to its base.

If homogeneous light be used, the light which passes through the prism will consist of two parts, diverging as if from points O_1 and O_2 symmetrically situated on opposite sides of the line CO. (§ 129.)

Suppose a sheet of paper to be placed at A, with its plane perpendicular to the line OCA, and let us consider what illumination will be produced at different parts of this paper. As O_1 and O_2 are images of O, crests of waves must be supposed to start from them simultaneously. Hence they will arrive simultaneously at A, which is equidistant from them, and there they will

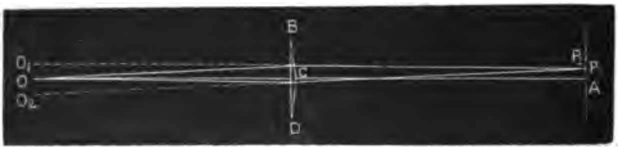

FIG. 41.

reinforce one another. Thus there will be a bright band on the paper parallel to the edges of the prism.

If P_1 be chosen so that the difference between P_1O_2 and P_1O_1 is half a wave-length (*i.e.* half the distance between two successive crests), the two streams of light will constantly meet in such relative conditions as to destroy one another. Hence there will be a line of darkness on the paper, through P_1, parallel to the edges of the prism.

At P_2, where O_2P_2 exceeds O_1P_2 by a whole wave-length, we have another bright band; and at P_3, where O_2P_3 exceeds O_1P_3 by a wave-length and a half, another dark band; and so on.

Hence, as everything is symmetrical about the bright band through A, the screen will be illuminated by a series of alternate parallel bright and dark bands, at approximately equal distances from one another.

230. If the paper screen be moved parallel to itself to or from the prism, the locus of all the successive positions of any one band will (by the nature of the curve) obviously be an hyperbola whose foci are O_1 and O_2. Thus the interval between any two bands will increase very nearly in proportion to the distance of the screen from the source of light.

But the intensity of the bright bands diminishes rapidly as the screen moves farther off; so that, in order to measure their distance from A, it is better to substitute the eye (furnished with a convex lens) for the screen. If we thus measure the distance AP_2 between A and the nearest bright band, measure also AO, and calculate (from the known material and form of the prism, and the distance CO) the distance O_1O_2, it is obvious that we can deduce from them the lengths of O_1P_2, and O_2P_2.

Their difference is the *length of a wave* of the homogeneous light experimented with. Though this is not the method actually employed for the purpose (as it admits of little precision), it has been thus fully explained here because it shows, in a very simple way, the possibility of measuring a wave-length.

231. And now we have a first hint of the extreme minuteness of the wave-period, for even a rude measurement, of the kind just described, shows that the wave-length of yellow light is somewhere about $\frac{1}{40000}$th of an inch only. In 186,000 miles this is contained somewhere about 500,000,000,000,000 times. This last

number must, therefore, approximately represent the number of waves of yellow light which in one second pass through each point on the ray.

232. The difference between O_1P_1 and O_2P_1 becomes greater as AP_1 is greater. Thus it is clear that the bands are *more widely separated the longer the wave-length of the homogeneous light employed.* [The positions of O_1 and O_2 change with the refrangibility of the light; but this change is slight, and does not seriously modify the result just given. Its tendency is to *increase* the effect.] Hence, when we use white light, and thus have systems of bands of every visible wave-length superposed, the band A will be red at its edges, the next bright bands will be blue at their inner edges and red at their outer edges.

But, after a few bands are passed, the bright bands due to one kind of light will gradually fill up the dark bands due to another; so that, while we may count hundreds of successive bright and dark bars when homogeneous light is used, with white light the bars become gradually less and less defined as they are farther from A, and rapidly merge into an almost uniform white illumination of the screen.

In this example, and in all others of a similar character which will be introduced into this elementary work, the solution is only *approximate.* The utmost resources of mathematics are in most cases required for the purpose of complete solution.

233. We are now in a position to prove that light moves slower in glass than in air, by the process which was merely *indicated* while we were discussing the speed of light. [See, again, § 71.]

For, if we could slightly lengthen the paths of the rays which come from O_1, leaving those from O_2 unaltered, the system of bands would obviously be shifted in the direction from A to P in the figure.

This happens if a very thin film of glass be interposed in the path of the rays which appear to come from O_1. The best mode of making the experiment is to put a piece of very uniform plate glass, cut into two parts, between the prism and the screen: so that rays from O_1 pass through one part, and those from O_2 through the other. So long as these pieces are parallel, no shifting takes place. But if *one* be slightly turned, so as to give the rays a longer path through it, the system of bands is at once displaced to the side at which it is situated.

234. Also, we can now see how it is theoretically possible to discover whether light has its speed affected by that of the medium in which it is travelling.

We know that sound travels faster *with* the wind, and slower *against* it, than it does in still air.

We may, therefore, suppose a disposition of the interference apparatus such that the two rays which interfere have each passed through a long tube full of water. A rapid current may be established, in either direction, in one or other of the tubes, or in opposite directions in the two, and the shifting of the interference-bands will at once indicate the nature of the effect. We cannot describe the details of the process. The result, however, is analogous to that of wind on sound, but of course very much smaller; and it seems that the actual change of the speed of light, thus produced, is *less* than the speed of the current. This has given rise to a theoretical discussion, of great importance, but quite unsuitable for this work.

235. Let us next consider the effect of a *grating;* a series of fine parallel wires placed at small equal intervals, or a piece of glass or of speculum metal on which a series of equidistant parallel lines have been ruled by a diamond point. We take only the case in which homogeneous light from a distant source falls perpendicularly on the plane of the grating, and when the bars and the openings of the grating are all equal in breadth.

FIG. 42.

236. Let ABCD be the plane of the grating. This line may, by our previous assumption, be also looked on as a section of one of the plane wave-fronts which fall on the grating.

Consider the effect produced on an eye or screen at a considerable distance, in the direction BE (fig. 42). If there were no grating, practically no light would reach the eye from the aperture AD unless ABE were very nearly a right angle. This is, of course, the statement of Huygens already quoted.

But Young's principle enables us to say *why* this is the case; to explain, in fact, what really becomes of Huygens' detached wavelets.

Let us divide AD into a series of equal parts, by lines perpendicular to BE, and distant from one another by half a wave-length of the homogeneous light employed. The portions coming to the eye from any two adjacent parts AB, BC, of the incident wave-front will be practically of the same intensity, and will exactly neutralise one another's effects on the eye. For if we take points *a* and *b*, similarly situated with regard to A and B respectively, the distances of *a* and *b* from the eye differ by half a wave-length, and rays from *a* neutralise those from *b*. This is true wherever *a* be taken between A and B. Hence, under the conditions assumed, no light reaches the eye.

[It will be noticed that, in what precedes, we have assumed that the aperture AD can be divided into a *number* of equal parts, such that their distances from E differ by half a wave-length. If this be not the case (*i.e.* if the aperture be of dimensions comparable with the wave-length) the above reasoning cannot be applied. Through a hole so small, light does not pass as a definite ray, but diverges in all directions behind the screen, just as sound does when passing through an aperture of a few feet in diameter. When sound passes through an aperture of several hundred feet or so in width, it is propagated approximately as a ray.]

237. Now suppose the alternate parts AB, CD, etc., to be opaque. Similar reasoning will show that the remaining rays conspire to strengthen one another.

Thus, when homogeneous light from a distant point falls perpendicularly on a grating in which the breadth of the bars is equal to that of the interstices, it will be seen brightly in a direction inclined at an angle θ (ABE) to the plane of the grating:—the angle θ being such that

$$AC \cos \theta = \text{wave-length}.$$

Similar reasoning shows that the light is reinforced whenever θ is such that

$$AC \cos \theta$$

is an integral multiple of the wave-length.

238. The appearance presented when a long narrow slit is the luminous object, and the bars of the grating are placed parallel to it, is therefore (with homogeneous light) a central image: with others, of rapidly diminishing intensity, equidistant from it on each side—their angular distances from it being the values of the angle corresponding to the sines

$$\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}, \text{ etc.}$$

Here λ is the wave-length, and a is the sum of the breadths of a bar and an interstice.

It is found in practice, and it is also deducible from the complete theory, that the *ratio* of the breadths of the bar and interstice has but little effect on the result (except, of course, as regards the whole amount of light transmitted), unless it be either very large or very small.

Hence if λ be expressed as a fraction of an inch, and n be the number of lines per inch in the grating, the angular deviations of the bright bands have the sines

$$n\lambda, 2n\lambda, 3n\lambda, \text{ etc.}$$

239. The mean wave-length of visible rays in air is about $\frac{1}{50000}$th of an inch. Thus a grating with 5000 equidistant lines per inch will give with such light an

angular deviation of about 6° (the angle whose sine is $\frac{5000}{50000}$, or $\frac{1}{10}$) for the first bright diffraction-line.

240. If we notice that the sine of the deviation is proportional to the wave-length, it will be obvious that when white light is used the result will be a series of spectra on each side of the central white image, their more refrangible ends being turned towards that image.

When the grating is a very regular one, and the appearances are examined by means of a telescope adjusted for parallel rays, the spectra formed in this way show the Fraunhofer lines with as great perfection as do the best prisms. And they have one special advantage, which prisms do not possess. The relative angular separation of the various colours depends solely on their wave-lengths, and thus the spectra formed by different gratings are practically similar to one another. There is, in fact, almost no *irrationality* in this kind of dispersion.

In glass prisms, and especially in those of flint glass, the more refrangible part of the spectrum is much dilated, while the less refrangible part is compressed.

241. The counting of the number of lines per inch in a grating is not difficult, nor is the accurate measurement of the angle of deviation of any particular Fraunhofer line.

Hence, by the help of the very simple formula given above, the wave-lengths of light corresponding to the various Fraunhofer lines have been determined with great accuracy from the diffraction spectra of gratings.

The following are, according to Ångström,[1] a few of

[1] *Spectre normal du Soleil*, Upsal, 1868.

the chief values. λ is expressed in ten-millionths of a millimètre.[1]

A		Atmospheric	7604	...
B		Atmospheric	6867	1·3309
C		Hydrogen	6562	1·3317
D	(double)	Sodium	{5895, 5889}	1·3336
E		Calcium and Iron	5269	1·3358
F		Hydrogen	4861	1·3378
G		Iron	4307	1·3413
H	(double)	Calcium and Iron	{3968, 3933}	1·3442

For the sake of a discussion to be entered on later, we have appended the refractive index from air into water for each of these rays, as given by Fraunhofer himself.[2]

242. If we suppose AB, CD, etc. (fig 42), to be transparent, while BC, etc., become opaque, it is obvious that the new grating will be the *complement* of the old one, and will give precisely the same appearances at points outside the course of the direct beam. For when there is no grating there is practically no illumination at such points, and therefore what passes in the first state of the grating is exactly capable of destroying, by interference, what passes through the grating in its second, or complementary state. This statement of course is equally true of any grating, whatever be the ratio of the breadths of the bars to those of the interstices.

243. Another very curious result of the theory of interference, fully verified by experiment, is furnished by the fact that the *central* spot of the shadow of a small circular disk, cast by rays diverging from a distant point

[1] As there are nearly 25 millimètres in an inch, these numbers, each multiplied by 4, give the wave-lengths approximately in thousand-millionths of an inch.

[2] Gilbert's *Annalen*, lvi., 1817.

in its axis, is as brightly illuminated as if the disk had not been interposed. The source of light should be an image of the sun, formed by a lens of very short focus; the disk a small circular piece of tinfoil pasted on glass (say half an inch in diameter). If this be placed 20 feet or so from the source, and its shadow received on a white screen 10 feet or so behind it, the phenomenon is easily exhibited.

244. The final example of interference which we can give here is noteworthy on account of a peculiarity which

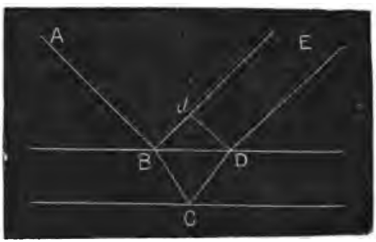

FIG. 43.

it presents. Let us consider the case of homogeneous light reflected by a thin uniform plate or film of a transparent isotropic material.

Let AB (fig. 43) be the direction of the incident ray, B*d*E the direction in which part of it is reflected to an eye E at a considerable distance; and let DE (of course parallel to BE) be the direction in which another part escapes, after refraction into the plate at B and partial reflection at the second surface of the plate at C. Then if D*d* be drawn perpendicular to BE, the retardation of the wave in DE as compared with that in BE will be $(2\mu BC - Bd)/\lambda$ wave-lengths, where μ is the refractive index into the plate.

But if α' be the angle of refraction, and t the thickness of the plate, it is easily seen that

$$BC \cos \alpha' = t,$$

and $$BD = 2BC \sin \alpha' = 2t \tan \alpha'.$$

Hence $$2\mu BC - Bd = 2\mu t \cos \alpha'.$$

245. Hence whenever, for a given thickness of plate, α' is such that

$$2\mu t \cos \alpha'$$

is an integral multiple of λ, the two rays should reinforce one another at E.

The same will happen for a given angle of incidence when the thickness of the plate is such that

$$2\mu t \cos \alpha'$$

is an integral multiple of λ.

When, on either account, $2\mu t \cos \alpha'$ is an odd integral multiple of $\lambda/2$, the rays at E will weaken (perhaps even destroy) one another.

From the simple expression or the retardation we see that, *cæteris paribus*, it is greater the greater the refractive index. It is less as $\cos \alpha'$ is less, *i.e.* as the obliquity of incidence is greater. For plates of the same material, the thickness must vary as $\sec \alpha'$ in order that they may give equal retardation.

246. Hence, in homogeneous light, a thin plate, turned about, alternately reflects and does not reflect to an eye in a given position. And a fixed plate of non-uniform thickness reflects light from some parts and not from others.

When white light is used there will in general be colours seen, which vary with the angle of incidence, and also with the thickness. All these results of the theory are at once verified by trial with thin films of blown glass, of split mica, or those of a soap-bubble. But we must remark that (as stated in § 205) the present investigation is incomplete. No account has, for instance, been taken of rays 3, 5, 7, etc., times reflected in the glass before their final escape to join the beam proceeding to E. These produce considerable modifications on the simple result above given.

It is interesting to examine, spectroscopically, the light thus reflected from thin plates. For we thus see at once, by dark *bands* in the spectrum, which portions are destroyed (by interference) in any direction.

247. If the plate is infinitely thin it would appear that there should be infinitely slight retardation only, and the plate should thus be bright in homogeneous light (and of course white in white light) at all incidences.

In general this is *not* the case. Thus when a soap-bubble, or a vertical soap-film, is screened from currents of air, and allowed to drain, the uppermost (*i.e.* the thinnest) part becomes perfectly *black*. It can, in fact, be seen only by the feeble light scattered by little drops of oil or particles of soap or dust on its surface.

Here, again, Young's sagacity supplied the germ at least of the explanation. It is given in the following extract from his *Theory of Light and Colours*, the Bakerian Lecture for 1801 already referred to:—

"PROPOSITION IV.—*When an undulation arrives at a Surface which is the Limit of Mediums of different Densities, a partial reflection takes place, proportionate in Force to the Difference of the Densities.*

"This may be illustrated, if not demonstrated, by the analogy of elastic bodies of different sizes. If a smaller elastic body strikes against a larger one, it is well known that the smaller is reflected more or less powerfully, according to the difference of their magnitudes: thus, there is always a reflection when the rays of light pass from a rarer to a denser stratum of ether, and frequently an echo when a sound strikes against a cloud. A greater body striking a smaller one propels it, without losing all its motion; thus, the particles of a denser stratum of ether do not impart the whole of their motion to a rarer, but, in their effort to proceed, they are recalled by the attraction of the refracting substance with equal force; and thus a reflection is always secondarily produced, when the rays of light pass from a denser to a rarer stratum. But it is not absolutely necessary to suppose an attraction in the latter case, since the effort to proceed would be propagated backwards without it, and the undulation would be reversed, a rarefaction returning in place of a condensation; and this will perhaps be found most consistent with the phenomena."

248. This idea, of a rarefaction returning by reflection when a condensation is incident, is equivalent to a loss or gain of half a wave-length when light in a denser body is reflected at the surface of a rarer body. Whether, then, the plate be denser or rarer than the medium surrounding it, one or other of the two interfering rays loses half an undulation relatively to the other in the mere act of reflection. This completely removes the difficulty.

But Young went further, and pointed out that if a thin plate be interposed between two media, one rarer, the other denser than the plate, this half wave-length effect should disappear. He verified this conjecture by direct experiment, founded on a modification of a process due to Newton.

249. Newton had, long before, devised and carefully employed an excessively ingenious (because extremely simple and effective) method of studying the colours of thin plates. It consisted merely in laying a lens of small curvature on a flat plate of glass. The film of air or other fluid between the spherical surface and its tangent plane has a thickness which is directly proportional to the square of the distance from the point of contact.

When such an arrangement is looked at in homogeneous light, the lens having been pressed into contact with the flat plate, there is seen a central black spot, surrounded by successive bright and dark rings, whose number appears to be practically unlimited. In accordance with the remark in § 245, the diameters of these rings increase with the obliquity of incidence of the light.

The radii of the successive bright rings were found by Newton to be as the square roots of the odd numbers 1, 3, 5, etc. Hence the thicknesses of the film of air are directly as these numbers. This is in accordance with the result of § 245, the lost half wave-length being allowed for. If water be placed between the glasses, instead of air, the rings are observed to be smaller. This also is consistent with the theory above. The interval between two successive rings rapidly diminishes from the central spot outwards. For it is obvious that the *areas* intercepted between successive rings are equal. Thus the interval varies very nearly as the reciprocal of the radius of either.

250. When rays of higher refrangibility are used, the

rings diminish in diameter. Hence when white light is employed we have a superposition of coloured rings of all sizes, but it is no longer possible to trace more than four or five alternations of *bright* and *dark* rings—the colours being then more and more compound.

This series of coloured rings is named after Newton, and the successive colours, gradually more and more composite, form Newton's *scale of colours.* Thus we read, in books more than thirty years old, of a red or blue of the *third order*, meaning those colours as seen in the third bright ring round the central dark spot.

251. Many of the most vivid colours of natural and artificial bodies are due to one or other of the forms of interference we have roughly explained.

Thus Barton's *buttons* (once employed for ornament, as they produce an effect very similar to that of diamonds) were simply polished metal plates stamped by a die of hardened steel, on whose surface had been engraved a pattern consisting of small areas ruled in different directions with close equidistant parallel grooves. Light reflected from such a surface behaves as if it had passed through a grating.

That the colours of a pearl and of mother-of-pearl are due to a similar surface corrugation, was proved by Brewster, who took impressions from such substances in black wax, and found that it was thus rendered capable of giving the same play of colours.

The scales from the wings of butterflies owe their bright colours to a delicate ribbed structure. On the other hand, the thin transparent wings of the house-fly, earwig, etc., owe their colours to their thinness. The same is true of the temper colour of steel, Nobili's rings, etc. Very beautiful examples of thin plates scaled off from decayed glass (found in Roman excavations) have been figured, with their play of colours, by Brewster.[1]

252. Here we can only say a word or two about the probable relation between the wave-length of homogeneous light and its refractive index for any isotropic medium. The existence of dispersion was attributed by Cauchy to the fact that even the most homogeneous media, such as water, have grained or heterogeneous structure, of dimensions not incomparably smaller than the average length of a wave of light. This grained structure has been recently proved to exist, by several perfectly independent processes suggested by totally unconnected branches of physics; and its dimensions have been assigned, at least in a roughly approximate manner.

253. It appears, from the theory of disturbances in such a medium, that the speed of a ray depends upon its wave-length in a manner which is expressed by a series of even inverse powers of that wave-length. Hence we have a relation such as

$$\mu = a + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^4} + \ldots\ldots$$

in which, from our present ignorance of the precise connection between matter and ether, we must be content to find the multipliers of the various terms by direct measurement.

If we neglect all but the first two terms, we may determine a and β from the known wave-lengths of two of Fraunhofer's lines, and their refractive indices for a particular medium. We can then test the accuracy of

[1] *Trans. Roy. Soc. Edin.*, 1861.

the formula by its agreement with the corresponding numbers in the same medium for others of the fixed lines.

Thus, taking the data for water given above (§ 241), we have, from the numbers for the two hydrogen lines C and F, the values

$$\alpha = 1{\cdot}3243,$$
$$\beta = 0{\cdot}00000000319.$$

Calculating from these, and the wave-length of H, we have for its refractive index 1·3447, instead of 1·3442 as determined by Fraunhofer.

So far as we may trust this theory, which certainly accords fairly with the experimental data for substances of moderate dispersive power, though by no means well with those for substances of high dispersive power such as oil of cassia, the value of the quantity α is the refractive index for the *longest possible waves; i.e.* it is that of the inferior limit of the spectrum. But the question of dispersion still remains very obscure, and will probably not be cleared up satisfactorily until we obtain much further information than we yet have as to the condition of the ether in the immediate neighbourhood of particles of ponderable matter, and the nature of the action between it and them.

254. Before we pass from the consideration of ordinary light, we must notice a remarkable consequence of the relative motion of the spectator and the luminous source, which depends upon the facts established in this and the preceding Chapters.

When a steamer at sea is moving in a direction perpendicular to the crests of the waves, she will encounter more of them in a given time if her course is towards them than if she were at rest, while, if she be moving in the same direction as the waves, fewer of them will overtake her in a given time than if she were at rest.

The same thing is true of sound-waves. When an express train passes a level crossing, at full speed, the pitch of the steam whistle is higher during the approach to, and lower during the recess from, the listener at the gate than it would be if the engine were at rest. The successive sound-pulses are emitted at the same intervals as they would be were the engine at rest, but from points successively nearer to or farther from the listener. Hence more or fewer reach his ear in a given time.

The principle when applied to light is usually associated with the name of Döppler, but it is precisely the same as that of Römer's observation of the frequency of the eclipse of Jupiter's satellites, which we have already given: the number of light-waves which reach the eye per second is increased if the source is approaching, and diminished if it be receding. The only difference is that we are now dealing with a phenomenon which occurs some 600,000,000,000,000 times per second, instead of once every forty-two hours.

Now, increased wave-frequency, with unaltered speed of light, certainly implies shorter wave-length, and most probably greater refrangibility, and *vice versâ*. In default of knowledge as to the true nature of the luminiferous medium, and of the species of vibration on which light depends, by which we might hope to be able to predict the result in any case, we must appeal to experiment. Observation has not yet settled the question of the relative motion of bodies, the ether they contain, and the ether in free space; but it has shown that something, quite analogous to the phenomenon above described with regard to sound, does take place with light.

This principle has been applied with success by Huggins and others to find the rate at which we are approaching to, or receding from, different fixed stars, and the rate of motion in solar cyclones; and it may even be applied, as was ingeniously suggested by Fox Talbot (*B. A. Report*, 1871), to determine (from the relative velocities of the components of a double star in the line of sight, measured by its aid) the distance of the star itself from our system.

CHAPTER XV.

DOUBLE REFRACTION AND POLARISATION.

255. We now come to phenomena which cannot be even roughly explained by processes based on the vague analogies of sound and water waves which have hitherto sufficed for our elementary treatment of the subject.

256. These phenomena were first observed in Iceland spar. They were described in a general way (§ 218) by Bartholinus, who showed that one of the two rays, into which a single incident ray is divided by this substance, follows the ordinary law of refraction.

257. Huygens, who studied the subject only eight years later, verified the greater part of the results of Bartholinus, and added many new ones. From his point of view it was of course obvious that the ordinary ray is propagated by spherical waves, *i.e.* its speed is the same in all directions inside the crystal. To explain the extraordinary ray, he assumed that it was propagated in waves of the form of an ellipsoid of revolution, the simplest assumption he could make.

258. To test this assumption he first noticed that a rhombohedral crystal of Iceland spar behaves in precisely the same way whichever pair of parallel faces light passes through. Hence he acutely concluded that the axes of

the ellipsoids of revolution (if such were the form of the waves for the extraordinary ray) must be symmetrically situated with regard to each of these planes. The only such lines in a rhombohedron are parallel to that which joins those corners which are formed by the meeting of three equal plane angles. In the case of Iceland spar these equal angles are obtuse. Huygens then verified, by experiments well contrived, though carried out by a very rough mode of measurement, the general agreement of his hypothesis with the facts; and he further tested it by comparing its indications, as to the position of the two images for any position of the crystal, with the results of direct observation. There can be no question that the whole investigation was, for the age in which it was made, of an exceedingly high order. But it must not be left unsaid that far more accurate measurements than those of Huygens were necessary before it could be asserted that the form of the extraordinary wave *is* an ellipsoid of revolution, and not merely a surface closely resembling such an ellipsoid. These improved measurements were made in 1802 by Wollaston, and they have recently been repeated with far more perfect optical means by Stokes, Mascart, and Glazebrook. The result has been the complete verification of Huygens' conjecture. The generating ellipse of the extraordinary waves is found to have its minor axis, which is that of revolution, equal to the diameter of the corresponding sphere for the ordinary ray. Its major axis is to the minor nearly in the ratio 1·654 : 1·483.

259. We are now in a position to trace the paths of the two rays into which a ray, falling in any direction on a surface of the crystal, is divided by refraction.

Let fig. 44 represent a plane wave-front AB (in air) falling on the surface AC of a piece of Iceland spar cut in any way. The figure is a section perpendicular to the surface, and parallel to the incident ray. The wave-front AB cuts the surface of the spar in a line (not shown) at right angles to the plane of the paper. Draw from

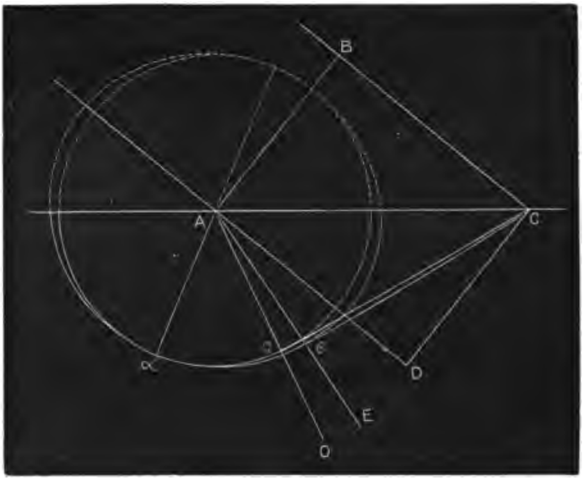

FIG. 44.

A the axis Aa of the crystal (not necessarily in the plane of the paper) and the sphere and ellipsoid of revolution which have Aa for a common semi-axis.

Then, if C be taken such that BC is to Aa as the speed of light in air is to that of the ordinary ray in the crystal, the wave-front of the ordinary ray is found by drawing a tangent plane to the sphere, passing through C and perpendicular to the plane of the paper. This touches the sphere in a point o (in the plane of the paper), and A*o*O is the ordinary ray. This is, of course, merely a repetition of the construction we have already given for singly refracting bodies (§ 215).

To find the direction of the extraordinary ray, a plane, perpendicular to the paper and passing through C, must be drawn so as to touch the ellipsoid. Let *e* be the point of contact, which will in general not be in the plane of the paper, unless A*a* is in or perpendicular to that plane; then A*e*E is the extraordinary ray.

Thus, in general, the extraordinary ray is not in the plane of incidence. Also the ratio of the sines of the angles of incidence and refraction is generally different for different directions of incidence, in the case of the extraordinary ray.

260. In an elementary work we cannot attempt to study these phenomena more fully; so we merely state that all the observed appearances, so far as the *directions* of the refracted rays are concerned, are explained by supposing the wave-surface in the crystal to be made up of the sphere and the ellipsoid of revolution above described. Thus when both eyes are used, the two images of a plane object seen through a crystal of Iceland spar appear in general to be situated at different distances above the plane. One of them maintains its apparent position as the crystal is made to rotate about a perpendicular to the two faces employed; the other's position varies as the crystal is turned.

261. But we have now to inquire *why* the incident ray is divided into two, and why one of them follows the ordinary law of refraction. Here another experimental result of Huygens comes to our assistance. We paraphrase the author's description:—

"I will, before concluding, mention another remarkable phenomenon which I discovered after the above was written. For, although I have not yet been able to find the cause of it, I do not wish on that account to refrain from pointing it out, in order that others may have an opportunity of seeking to explain it. It appears that it will be necessary to make hypotheses additional to those already given,—though these will lose none of their probability, confirmed as they have been by so many tests. The phenomenon is that, taking two fragments of the crystal (Iceland spar) and laying them on one another, or even holding them apart, if all the faces of the one be parallel to those of the other, a ray of light divided into two by the first fragment will not be further subdivided by the second. The ordinary ray from the first will be refracted ordinarily by the second, the extraordinary ray extraordinarily. And the same thing happens not only in this arrangement but in all others in which the principal sections[1] of the two fragments are in the same plane, whether the surfaces turned towards one another be parallel or not. It is, in fact, marvellous that these rays, falling on the second fragment, do not divide like the ray incident on the first. One would say that the ordinary ray from the first fragment *had lost* what is necessary for the production of extraordinary refraction, and the extraordinary ray that which is necessary for ordinary refraction; but there is something else which upsets this view. For when one places the fragments so that their principal sections are

[1] Defined as passing through the shorter diagonal of one of the rhombic faces of the crystal, and through the edge formed by the two adjacent faces.

at right angles, whether the opposed surfaces be parallel or not, the ordinary ray from the first suffers only extraordinary refraction by the second, and *vice versâ.*

"But in all the infinite number of positions other than those named, both rays from the first fragment are divided into two by the second. Thus the single incident ray is divided into four, sometimes equally, sometimes unequally bright, according to the varying relative position of the crystals. But all together do not seem to have more light than has the single incident ray.

"When we consider that, the two rays given by the first crystal remaining the same, it depends upon the position of the second crystal whether they shall be divided into two or not, while the incident ray is always divided, it appears that we must conclude that the waves of light which have traversed the first crystal have acquired a form or disposition which in some positions enables them to excite the two kinds of matter which give rise to the two kinds of refraction, in other positions to excite only one of them. But I have not yet been able to find any satisfactory explanation of this."

So far Huygens. His statements are perfectly in accordance with fact; and they were reproduced by Newton[1] in very nearly the same form. Newton adds:—"The unusual refraction is, therefore, performed by an original property of the rays. And it remains to be inquired, whether the rays have not more original properties than are yet discovered. Have not the rays of light several sides, endued with several original properties?"

262. It is very curious to notice how near each of

[1] *Optics*, Queries 25, 26.

these great men came to the true explanation, and yet how long time elapsed before that explanation was found. The date of Huygens' work is 1690, that of Newton's 1704. It was not till 1810 that further information on the subject was obtained. *Then* one brilliant observation opened the way for a host of discoveries in a new and immense field of optics.

263. In the last-mentioned year Malus, while engaged on the theory of double refraction, casually examined through a doubly refracting prism the sunlight *reflected* from the windows of the Luxembourg palace. He was surprised to find that the two rays alternately disappeared as the prism was rotated through successive right angles, —in other words, that the reflected light had acquired properties exactly corresponding to those of one of the rays transmitted through Iceland spar. Even Malus was so imbued with the corpuscular theory of light that he named this phenomenon *polarisation*, holding it as inexplicable on the wave theory, and as requiring a species of polarity (akin to the magnetic) in the light-corpuscles—a close reproduction of one of Newton's guesses.

264. But, after a short time, Hooke's old guess was independently reproduced, and in the hands of Young and others, but most especially of Fresnel, the consequences of the assumption, that the vibrations of the luminiferous medium take place *perpendicularly* to the direction of the ray, were the almost complete explanation of the cause of double refraction, and the discovery (often the prediction) of a long series of the most gorgeous phenomena known to science.

265. The real difficulty in the way of this conception

probably lay in the fact that most of the familiar forms of wave-motion—such as sound-waves in air or in water, and ordinary water-waves—are not of this character. In sound-waves the vibrations are wholly in the direction of the ray, while in surface-waves in water they are partly parallel to, and partly perpendicular to, the direction in which the wave is travelling. That a body may transmit waves in which the vibration is perpendicular to the direction of a ray, it must have the properties of an elastic *solid* rather than of a fluid of any kind. And our experience of the almost entire absence of resistance to the planetary motions seems, at first sight at least, altogether incompatible with the idea that the planets move in a jelly-like solid, filling all space through which light can be propagated.

266. Without going into difficult dynamical details, we may obtain a notion of the nature of the motion now to be considered, by observing the propagation of a wave when a long stretched wire or string is struck or plucked near one end. Here the line of motion of each part of the wire is almost exactly perpendicular to the direction of the wire, *i.e.* to the line along which the wave travels. (When the string is extensible there may be another wave, due to extension; but this, which is analogous to sound, has its vibrations *along* the string, and it usually travels at a very different rate from the other, so that the two are not in any way associated.)

267. Now it is clear that waves of this wholly transverse character can have, in Newton's language, *sides*. And it is also clear that they cannot interfere so as mutually to destroy one another, unless their corresponding sides are parallel to one another; nor can they

interfere *at all*, so as to modify one another's intensity, if their sides are perpendicular to one another. Hence a very severe test of the theory will be furnished by examining various cases of interference of polarised light, which ought to present in general marked differences from those of ordinary light. It was by experiments of this kind that Fresnel and Arago first firmly established the bases of the theory of polarisation.

268. The important fact discovered by Malus was soon generalised into the following statement:—

Light reflected from the surface of substances so different as water, glass, polished wood, etc., at a certain definite angle, which depends on the nature of the substance, is found to possess all the properties of one of the rays transmitted through Iceland spar. If the plane of reflection is parallel to the axis of the spar, the properties of the reflected light are those of the *ordinary* ray; if perpendicular to it, those of the *extraordinary* ray.

269. It was reserved for Brewster to discover, as the result of a very laborious series of experimental measurements, the simple law which follows:—

The tangent of the polarising angle is equal to the refractive index of the reflecting substance.

This may be put in another form, in which its connection with theory is a little more evident:—

When the reflected ray is completely polarised, it is perpendicular to the refracted ray.

270. Bearing in mind Huygens' observations on light which has passed through two crystals of Iceland spar, we can now see that a ray of light polarised by reflection is in general divided into two by a crystal of Iceland spar. But there is only one ray when the prin-

cipal plane of the crystal is parallel to the plane of reflection, and also when these planes are perpendicular to one another.

271. We may now much simplify matters by suppressing the Iceland spar, and using two reflecting plates of glass, so placed that a ray meets each of them in succession at the polarising angle. It is then found that when the planes of reflection are parallel the ray is reflected (almost without loss) from the second plate, but when they are perpendicular to one another there is complete extinction. In intermediate positions the intensity was found by Malus to be as the square of the cosine of the inclination of these planes.

This very simple experiment, which any one may easily make for himself, by putting two pieces of glass (blackened at the back) at the proper angle in the ends of two wooden tubes which fit into one another, enables us to form a general notion of the modification which is called polarisation. The "sides" of the reflected ray are obviously in, and perpendicular to, the plane of incidence; for a ray can be reflected over and over again if the successive planes of incidence are parallel, but is stopped at once if one of them be perpendicular to the others.

272. Here, however, two new difficulties come in at once:—

(1) Are the vibrations of the reflected ray in, or perpendicular to, the plane of reflection?

(2) As ordinary sun- or lamp-light, reflected at the proper angle from a polarising surface, shows no variation of intensity when the azimuth of the plane of reflection is changed, what can be then the direction of *its* vibrations?

273. Many important phenomena are explained in terms quite independent of the proper answer to (1); and in others which do depend on the answer the theoretical differences between the results of the two hypotheses are so small as to have hitherto remained undetected.

When we think of Huygens' result (§ 261) in connection with that of Malus (§ 268), we see that if the vibrations of light polarised by reflection be *perpendicular* to the plane of reflection, those of the ordinary ray in Iceland spar will be perpendicular to the axis of the crystal, which is an axis of optical symmetry. Hence we find no difficulty in accounting for the fact that the ordinary ray is propagated with equal speed in all directions in the crystal. If we assumed the vibrations to be *parallel* to the plane of reflection, we should find great difficulty in explaining the behaviour of the ordinary ray in Iceland spar.

Haidinger[1] strengthened this species of argument by an examination of the behaviour of *dichroic* uniaxal crystals, such as tourmaline. These colour both rays in the same manner, when they pass nearly in the direction of the axis, and, therefore, necessarily have their vibrations perpendicular to it. At greater inclinations to the axis, the ordinary ray preserves its colour, but the extraordinary ray changes colour in a marked manner. Thus we naturally conclude that the ordinary ray is, in all cases, due to vibrations perpendicular to the axis, and, therefore, to its plane of polarisation.

In one important test, suggested by Stokes,[2] the

[1] *Pogg. Ann.* lxxxvi. 131 (1852).

[2] "On the Dynamical Theory of Diffraction," 1849. See Stokes' *Math. and Phys. Papers*, vol. ii. p. 327.

results of different experimenters have been at variance in a way not yet thoroughly explained.

Another test, however, also due to Stokes,[1] appears to settle the matter. In the passages which follow, the term "false dispersion" is employed to signify "light reflected from motes." (See, again, § 199.)

"If a little tincture of turmeric be greatly diluted with alcohol, and then water be added, a yellow fluid is obtained which appears to be perfectly clear, exhibiting no sensible opalescence; but the occurrence of a copious false dispersion, when the fluid is examined by sunlight, reveals at once the existence of suspended particles, though they are too minute to be seen individually, or even to give a discontinuous appearance to the falsely dispersed beam."

"When a horizontal beam of falsely dispersed light is viewed from above, in a vertical direction, and analysed, it is found to consist chiefly of light polarised in the plane of reflection. It has often struck me, while engaged in these observations, that when the beam had a continuous appearance, the polarisation was more nearly perfect than when it was sparkling, so as to force on the mind the conviction that it arose merely from motes. Indeed, in the former case, the polarisation has often appeared perfect, or all but perfect. It is possible that this may in some measure have been due to the circumstance, that when a given quantity of light is diminished in a given ratio, the illumination is perceived with more difficulty when the light is uniformly diffused than when it is spread over the same space, but collected into

[1] "On the Change of Refrangibility of Light," *Phil. Trans.*, p. 530. 1852.

specks. Be this as it may, there was at least no tendency observed towards polarisation in a plane perpendicular to the plane of reflection, when the suspended particles became finer, and therefore the beam more nearly continuous.

"Now this result appears to me to have no remote bearing on the question of the direction of the vibrations in polarised light. So long as the suspended particles are large compared with the waves of light, reflection takes place as it would from a portion of the surface of a large solid immersed in the fluid, and no conclusion can be drawn either way. But if the diameters of the particles be small compared with the length of a wave of light, it seems plain that the vibrations in a reflected ray cannot be perpendicular to the vibrations in the incident ray. Let us suppose for the present, that in the case of the beams actually observed, the suspended particles were small compared with the length of a wave of light. Observation showed that the reflected ray was polarised. Now all the appearances presented by a plane polarised ray are symmetrical with respect to the plane of polarisation. Hence we have two directions to choose between for the direction of the vibrations in the reflected ray, namely, that of the incident ray, and a direction perpendicular to both the incident and the reflected rays. The former would be necessarily perpendicular to the directions of vibration in the incident ray, and therefore we are obliged to choose the latter, and consequently to suppose that the vibrations of plane polarised light are perpendicular to the plane of polarisation, since experiment shows that the plane of polarisation of the reflected ray is the plane of reflection.

According to this theory, if we resolve the vibrations in the incident ray horizontally and vertically, the resolved parts will correspond to the two rays, polarised respectively in, and perpendicularly to, the plane of reflection, into which the incident ray may be conceived to be divided, and of these the former alone is capable of furnishing a reflected ray,—that is, of course, a ray reflected vertically upwards. And in fact observation shows, that, in order to quench the dispersed beam, it is sufficient, instead of analysing the reflected light, to polarise the incident light in a plane perpendicular to the plane of reflection.

"Now in the case of several of the beams actually observed, it is probable that many of the particles were really small compared with the length of a wave of light. At any rate they can hardly fail to have been small enough to produce a tendency in the polarisation towards what it would become in the limit. But no tendency whatsoever was observed towards polarisation in a plane perpendicular to the plane of reflection. On the contrary, there did appear to be a tendency towards a more complete polarisation in the plane of reflection."

This beautiful experiment, and the reasoning based upon it, show at once how to account for the fact that the light of the sky is polarised. The reader who wishes to know the facts of atmospheric polarisation should consult the valuable paper by Brewster (*Trans. R. S. E.*, 1863), where a long-continued series of observations is given.

274. It is quite possible that, as is required by Clerk-Maxwell's *Electro-magnetic Theory* of light,[1] there may be simultaneous displacements, but of different characters,

[1] See his *Electricity and Magnetism*, Chap. XX.

in each of these planes, and then the question would be reduced to—Which of these displacements is the luminous one? But on this theory, *both* are probably essential to vision, as they are essential to wave-propagation.

275. As to the second question, it may be said—*first*, that, so far as the test of double refraction can inform us, a polarised ray, *whose plane of polarisation is made to rotate very rapidly* (whether uniformly or not), produces precisely the same effects as a ray of ordinary light; and, *secondly*, that, so great is the number of vibrations even of red light in one second, it would be impossible to make the plane of polarisation rotate fast enough to affect the circumstances of any of the phenomena of interference, even when they take place between two portions of the same ray, one of which is retarded thousands of wave-lengths more than the other. But, *thirdly*, the fact that, when homogeneous light is used, Newton's rings have been *counted* up to the 7000th, shows that, whatever be the actual nature of the vibrations of unpolarised light, they must for at least 7000 waves in succession be almost precisely similar to one another. Then for other 7000 waves or so we may have a totally different type of vibration. This is the suggestion made by Airy, in his *Tract* on the *Undulatory Theory*. But, *fourthly*, in the course of $\frac{1}{7}$th of a second, at the very utmost, the vibrations must have been almost uniformly distributed over all directions perpendicular to the ray; else interference phenomena, such as those described in § 283 below, would exhibit constant changes of appearance although no part of the apparatus employed were altered in position.

Here, however, Stokes again comes to our assistance.

In his paper, entitled "On the Composition of Streams of Polarised Light from different Sources,"[1] he shows what will be the *average* effect of a very great number of special sources of light; thus giving one of the earliest illustrations of the use of the statistical methods of *Probabilities* in physics. We can here merely give an idea of the *nature* of the explanation.

All common light has its origin from a practically infinite number of sources, consisting of the vibrating particles of the luminous body. The contributions from each of these sources (so far as one definite wave-length is concerned) which are received at any one point may be, and probably *are*, as different in direction of vibration as they certainly must be in phase;[2] and therefore, in order to estimate the total effect at any instant at that particular point, we must apply the methods of averages. From this point of view the uniformity of optical phenomena becomes quite analogous to the statistical species of uniformity which is now found to account for the behaviour of the practically infinite group of particles forming a cubic inch of gas. The reader need only think of the fact that, so numerous are those particles, it is practically (though not theoretically) impossible that even a cubic millimètre of air should, even for $\frac{1}{10000}$th of a second, contain oxygen particles alone.

276. When light is reflected at an incidence either less or greater than the polarising angle, it behaves as if part of it only were polarised and the rest ordinary light; and it is said to be *partially polarised*. Tested by a

[1] *Camb. Phil. Trans.*, 1853.

[2] A curious exception occurs in the case of light radiated from a body which polarises by absorption. See Chap. XVI.

crystal of Iceland spar, it gives two images in all positions of the crystal; but their brightness is unequal except in the special positions where they would be of equal brightness were the ray wholly polarised.

277. From the fourth of the remarks made above regarding common light, and the facts of double refraction, it follows at once that, when light is to any extent polarised by reflection, there must be an exactly equal amount of polarised light in the refracted ray, and its plane of polarisation must be perpendicular to the plane of refraction. This was established by experiment soon after Malus' discovery. But as the reflected ray from glass, water, etc., is in general much weaker than the refracted ray, the percentage of polarised light is generally much greater in the former. It was found, however, by experiment that refraction at a second glass plate, parallel to the first, increases the proportion of polarised to common light in the transmitted ray, and thus that light may be almost completely polarised by transmission, at the proper angle, through a number of parallel plates. The experimental data of this subject were very carefully obtained by Brewster. He has found, for instance, how the angle of incidence for the most complete polarisation varies with the number of plates. The plane of polarisation of such a bundle is *perpendicular* to the plane of refraction.

278. This, however useful on many occasions, is at best a rough arrangement for producing polarised light. By far the most perfect polariser for a broad beam of light is a crystal of Iceland spar, sufficiently thick to allow of the complete separation of the two rays. But such specimens are rare and costly, so that the polariser

in practical use is now what is called *Nicol's prism*, invented in 1828.[1] By cutting a rhomb of Iceland spar in two, and cementing the pieces together with Canada balsam (after carefully polishing the cut faces), Nicol produced an arrangement in which one only of the two rays is transmitted, the other being totally reflected at the surface of the balsam. The reason is that the refractive index of Canada balsam is intermediate between those of the ordinary and extraordinary rays in the spar. The ordinary ray in the spar, falling very obliquely on a medium of a smaller refractive index, is totally reflected; the extraordinary ray, falling on a medium of greater, but very little greater, refractive power, is almost wholly transmitted. The only defect of the Nicol's prism is that, to secure the total reflection, its length must be considerably greater than its breadth; and thus it necessarily limits the divergence of the beam it allows to pass.

279. Certain doubly refracting crystals exert considerable absorption on one of the two rays they produce, and can therefore, when in plates of sufficient thickness, be employed as polarisers. This is the case with some specimens of tourmaline when cut into plates parallel to the axis of the crystal. It is also found in the flat crystals of several artificial salts, such as, for instance, iodo-sulphate of quinine. (See, again, § 273.)

280. Let us now suppose that by one or other of these pieces of apparatus, say a Nicol's prism, light has been polarised. If we examine this ray by means of a second Nicol, placed in a similar position to the first, it passes practically unaltered. As the second Nicol is made to rotate, more and more of the light is stopped,

[1] *Jameson's Journal*, p. 83.

till the rotation amounts to a right angle. Two well-constructed Nicols, placed in this position, are practically opaque to the strongest sunlight. During the next quadrant of rotation the transmitted ray gradually increases in brightness, until at 180° of rotation it passes practically unaltered. Precisely the same phenomena occur in the same order during the next half of a complete rotation. The reader will observe that this is merely Huygens' original statement, limited to one of the four rays which are produced by passing common light successively through two crystals of Iceland spar.

A Nicol, therefore, enables us to test directly what portions of a beam of light are polarised in a particular plane. And the rays which pass through a Nicol, in any one position, and in a second produced by rotating it either way through a right angle, are strictly complementary, in the sense that, if superposed, they would reproduce the original beam.

281. Whatever be the true mechanism of polarised light, there can be no doubt that its vibrations are *symmetrical* with respect to the ray, and also with respect to the plane of polarisation. Hence we may, for many important purposes, symbolise them by simple harmonic vibrations taking place either in or perpendicular to the plane of polarisation. But, if they be supposed to take place simultaneously in these two planes, their quality or nature must be essentially different in the two, else the symmetry above referred to would be violated. Hence it will be sufficient for the present to assume that they take place perpendicular to the plane of polarisation. The nature of the resulting effects, so far as the eye is concerned, will not be different for the different hypo-

theses. Also, as no instance has yet been observed, even with the most intense beams of light, in which the joint effects produced are not those due to simple superposition, we may assume that the elastic force of the luminiferous medium, called into play by a displacement, is directly proportional to the displacement, and therefore that the vibrations for each wave-length follow the *simple harmonic law*, that of the cycloidal pendulum.

282. The subject of the composition of simple harmonic motions (*i.e.* vibrations such as those of a tuning-fork, or of an ordinary pendulum vibrating through very small arcs) of equal period is an important branch of *Kinematics*, upon which we cannot here dilate. We will therefore simply assume the following results:—

1. Two simple harmonic motions of the same period, in lines perpendicular to one another, give in general, when superposed, elliptic motion such as that of a simple pendulum slightly disturbed, and not vibrating in one vertical plane. This motion may be in the positive or negative direction of rotation.

2. The ellipse becomes a straight line, and the resultant motion therefore simple harmonic, when the phases of the components are the same, or differ by an integral multiple of π.

3. It becomes a circle when the amplitudes of the components are equal, and their phases differ by an odd multiple of $\frac{1}{2}\pi$. The motion takes place in one direction (say right-handedly) in the circle when this multiplier is 1, 5, 9, 13, etc., and in the opposite (left-handed) when it is 3, 7, 11, 15, etc.

These statements may be verified in an extremely easy and highly instructive manner, by means of Blackburn's pendulum. Let AC, CB, fig. 45, be two fine wires, of equal length, attached to points A, B in the same horizontal line. For convenience in describing the results, suppose that A lies due north of B. Let a third and (for our present purpose) very much longer wire, CD, be attached to the others at C, and support D, the bob of the pendulum.

When the motion of D is wholly north and south, it behaves as a simple pendulum of length CD. But, if its motion be wholly east and west, the whole system of wires turns like a rigid body about the line AB. Hence D moves as a simple pendulum, of length greater than CD by the distance of C from AB. The period of vibration is therefore slightly longer in the E.W. than in the N.S. direction. [The ratio of the periods may be altered to any extent by varying the relative lengths of the wires, and the distance AB.] When D is displaced through a small angle in a direction neither N.S. nor E.W., the displacement may be resolved into components in these directions, and the motion of D corresponds to a *superposition* of the motions due to these component displacements separately. Hence D has a definite range of excursion N. and S., and another definite range E. and W.; so that, whatever be its path, it must always *touch* in succession the sides of a certain rectangle.

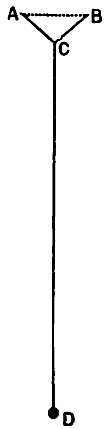

FIG. 45.

In fig. 46, let O represent the equilibrium position of the pendulum bob; Oy a northward, and Ox an eastward line; and OA the initial displacement of the bob.

Produce AO to B, where OB = AO, and construct the parallelogram AB'BA', whose sides run N.S. and E.W. Then the motion of the bob consists of a pendulum motion of range NN', superposed on another of range MM' whose period is slightly longer. And, since the whole original displacement was OA, the pendulum was (at starting) at N in the motion NN', and at M in the motion MM'.

When the N.S. pendulum has arrived at O, in its course NON', the E.W. pendulum has not quite reached O. Hence in the complex motion the bob, displaced to A, will (when let go) pass to the *right* of O, touch A'B and then BB', each at a point near to B, pass to the left of O, and so on. Its motion is therefore, at each instant, in an ellipse in which it is moving negatively or clock-wise.

This ellipse gradually widens out, until it becomes the greatest ellipse which can be inscribed in the rectangle, *i.e.* that whose principal semi-axes are ON and OM. Here the N.S. pendulum has gained a quarter oscillation on the other, for it is at N (the extremity of its range) while the E.W. pendulum is at O (midway in its range).

The ellipse now gradually narrows again, as its point of contact with BA' passes from N' towards A', and becomes the diagonal A'B' when the N.S. pendulum has gained half an oscillation on the other.

But, in returning from B', the pendulum bob will pass to the *left* of O, because NO is described in less time than M'O. The motion again becomes elliptical, going through the same set of ellipses as before—but in the opposite order. And the bob is now moving positively or counter-clock-wise in its ellipse.

Permanent records of the whole track of the bob may be made, on a horizontal sheet of paper placed close below the pendulum, by a fine stream of ink escaping from the bob; or, still better, by sparks from an induction coil, one pole of which is connected to one of the points of suspension of the pendulum, the other to a plate of metal on which the paper is laid.

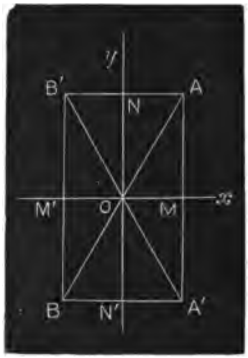

FIG. 46.

If we fix the point C (fig. 45), at any instant when it is in the same vertical plane with A and B, the pendulum will describe (completely, and for an indefinite period) the *instantaneous* ellipse of which it was at that instant describing a portion.

283. Now suppose a plane polarised ray of one definite wave-length, *i.e.* homogeneous light, to fall on a plate of a doubly-refracting crystal (a thin plate of mica or selenite, for instance). Within the plate it will in general be divided into two, which are polarised in planes at right angles to one another. The directions of vibration in these rays are determined by the physical properties of the material. Let them be represented by the lines Ox, Oy in fig. 46. Then, if OA represents the semiamplitude of vibration in the incident ray, it may be looked on, by § 282, (2), as the resultant of two simple harmonic motions of the same period, whose semiamplitudes are OM and ON, and which are in the same phase.

Each of these will pass through the plate of crystal *unchanged*. But one will, in general, travel faster than the other; for one of the essential causes of double refraction is the difference of speeds of the two rays. The portions of the two rays which simultaneously escape from the crystal, and which travel together outside it (parallel to their path at incidence), will therefore *differ in phase*. Hence, to find the nature of the transmitted light, we must recombine the vibrations in OM, ON, taking account of this difference of phase. By § 282, (1), the result will be in general elliptic motion. The ellipse will necessarily be one of the infinite number which can be inscribed in the rectangle AA′BB′, whose construction is obvious. For the *amplitudes* of the excursions parallel to Ox and Oy are not altered; but the maximum displacement in the one direction is, in general, no longer *simultaneous* with that in the other. We have then, in general, what is called *elliptically polarised light*. This degenerates, by § 282, (2), into plane polarised light, whose vibrations are along OA or OA′ according as the difference of phase is 0, 2π, 4π, etc., or π, 3π, 5π, etc. And it will become *circularly polarised light* if OM = ON (*i.e.* if AO$x = \frac{1}{4}\pi$) and the difference of phase be an odd multiple of $\frac{1}{2}\pi$. By § 282, (3), this will be right or left handed, according to the value of the odd multiplier.

Fig. 46.

284. This conclusion from the assumption above made is fully borne out by experiment. When a plate of mica, of such a thickness as to retard one of the two rays a quarter of a wave-length more than the other, is interposed between two Nicols, we observe the following phenomena:—

If the Nicols were originally placed so as to extinguish the light, the introduction of the mica plate in general partially restores it. Now, let the mica plate be made to rotate in its own plane. The light vanishes for successive positions, differing by a quadrant of rotation, *i.e.* whenever the directions of vibration in the crystal coincide with the principal planes of the Nicols. In each of these positions the light from the first Nicol passes unchanged (except in phase) through the mica, and is therefore entirely stopped by the second Nicol.

Half-way between these positions the light transmitted through the system is at its brightest; and in such a case it is not altered in brightness by rotating the second Nicol. It is then *circularly* polarised; and in whatever position the second Nicol is placed, the component of the circular motion which is ready to pass through it is of the same amplitude. Here, then, is a case in which a Nicol (the second) cannot enable us to distinguish between common light and light very seriously modified.

If the Nicols be placed in any (random) relative position, the introduction of any doubly-refracting plate will, in general, affect the intensity of the transmitted light. And, if the plate be turned round in its own plane, it will produce an effect depending on the rotation. This effect is null in every one of the positions in which *either* of the directions of vibration in the plate is perpendicular to the plane of polarisation of the light transmitted by *either* Nicol.

285. In what precedes, we have assumed that *homogeneous* light was used. In general, a doubly-refracting plate produces a difference of phase in its two rays whose amount depends on the wave-length; and thus when white light is used we have a display of colour, sometimes extremely gorgeous: and we may distinguish light, circularly polarised as in § 284, from common light, by slight changes of colour and intensity as the second Nicol is turned.

286. Hitherto we have spoken of the polarising angle for light reflected in air from bodies such as glass, water, etc., which have a higher refractive index than air, and we have seen that an equal amount of light is polarised in the refracted beam. But what if there be no refracted beam? This is the case of total reflection inside the denser body.

Fresnel discovered that in this case the two kinds of polarised light (in planes at right angles to one another) co-exist in the totally reflected ray, but that they differ in phase, and therefore in general recombine into elliptically polarised light. Guided by peculiar theoretical considerations, he was led to construct a piece of glass (*Fresnel's rhomb*), inside which light is twice totally reflected at a certain angle (depending on the refractive index of the glass), with the result that, if it be originally polarised in a plane inclined at 45° to the plane of reflection, the emergent light is circularly polarised.

287. Reflection from the surface of metals, and of very highly refractive substances such as diamond, generally gives at all incidences elliptically polarised light.

Attempts have been made to determine from such effects the refractive indices of metals and other opaque substances. These are all based upon theory, and cannot as yet command much confidence. With certain doubly-refracting substances the light reflected at a definite angle is differently polarised, and sometimes even differently coloured, for different azimuths of the plane of incidence. Substances which exert powerful absorption on definite portions of the spectrum exhibit, by reflection, a quasi-metallic lustre. This is beautifully shown by many of those aniline compounds (§ 197) which produce abnormal dispersion.

288. When a thin plate of doubly-refracting crystal, which gives a bright colour when placed between two Nicols, is slightly inclined to the ray, the colour changes as the difference of phase of the two refracted rays is increased. If, now, we take a plate of Iceland spar cut perpendicularly to the axis, no colour will be produced by parallel rays passing through it perpendicularly, because both rays have a common speed parallel to the axis; but, if convergent or divergent light be used, there is a gorgeous display of circular coloured rings surrounding the axis, which depends upon the increasing retardation of the ordinary ray behind the extraordinary as their inclination to the axis increases. When the principal planes of the Nicols are at right angles, this system of rings is intersected by two black diameters, in these planes respectively. When the second Nicol is turned through a right angle, we have exactly the *complement* of the former appearance, *i.e.* a figure such that, if superposed on the former, it would give an uniform field of white light (§ 280).

289. It is to be noticed that none of these phenomena can be observed without the use of the second Nicol. This arises from the fact that, where the vibrations in any direction interfere so as to weaken one another, those in the direction perpendicular to the former interfere so as to strengthen one another to an equal amount. The second Nicol enables us to select one of these portions, and examine it independently of the other.

[There are individuals, generally with very dark eyes, who are able to distinguish polarised light from common light, because of a polarising structure in the eye itself. This gives rise to what are called *Haidinger's Brushes*, whenever polarised light falls on the eye. Such individuals see the brushes in *all* reflected or refracted (*i.e.* partially polarised) light. The great majority of men, however, can only see the phenomenon with polarised light, and then with difficulty. The best way of making the observation is to look through a Nicol at a bright cloud, or a piece of white paper well illuminated, and to give a slight rotation to the Nicol at intervals. In the line of sight there will be detected four little coloured tufts, or brushes, two having a brownish, the others a bluish or purplish tint.]

290. The only double refraction we have considered particularly is that of Iceland spar, where everything is symmetrical about the axis of the crystal. Such crystals, and they include as a rule all those of the second and third crystallographic systems, are called *uniaxal*. Crystals of the first system are not doubly refractive.

But it was one of the most valuable of Brewster's discoveries that the great majority of non-isotropic substances are doubly refracting, and in general are *biaxal*, *i.e.* have two equally important *optic axes*, whose mutual inclination may have any value from 0° to 90°. The form of the *wave-surface* in such bodies was, at least very approximately, assigned by Fresnel. This forms one of the most brilliant of his many grand discoveries; and it led to Hamilton's prediction of the existence of the two species of *conical refraction*, which was experimentally verified by Lloyd.

291. According to Fresnel's approximate theory, the form of the wave-surface is now no longer the sphere and rotation-ellipsoid of Huygens, but a single surface of the fourth order, consisting of an inner and an outer sheet which meet at four points, through which the one passes continuously into the other.[1] A small portion of the surface in the neighbourhood of one of these points forms approximately a double cone of large angle, of which the point is the vertex. One of the two cones is tangent to the outer, the other to the inner, sheet of the wave. The appearance of the outer sheet, in the neighbourhood of one of these points, closely resembles that of the part of an apple round the point of insertion of the stalk. And, like the apple, it can be touched by a tangent plane along a circle surrounding the dimple.

Now if we make the construction of § 259 with a sur-

[1] Fresnel's elegant construction of this surface is as follows:—Draw any central section of an ellipsoid, and a line through the centre perpendicular to the plane of the section Along this line lay off, in either direction from the centre, lines equal to the chief semi-axes of the section. Their extremities lie on the wave-surface. The form of the ellipsoid, in any case, depends on the nature of the crystal.

face like this, it is obvious that in general there will be a single tangent plane to the outer, and another to the inner, sheet of the surface. Thus we get the two refracted rays. But it is only under special conditions that *either* of them will be in the plane of incidence. Hence in biaxal crystals both rays are extraordinary.

But, for one special direction of the incident ray, the tangent plane will touch the surface along the circle above spoken of. Hence there will be an infinite number of refracted rays, forming a cone of which the circle is the base. If the sides of the plate of crystal be parallel, each of these rays will escape parallel to its direction at incidence, and thus the single incident ray will emerge as a *hollow cylinder*.

But by using small holes in thin plates of metal, placed on the surfaces of the plate of crystal, we may prevent any light from passing through it except along the line joining the centre of the wave-surface with one of the conical points. The incident rays corresponding to this will form a hollow cone, and will emerge again as a hollow cone; because at the conical point there is an infinite number of tangent planes. Hence, for the proper exhibition of this phenomenon, the light should be convergent at incidence. Sun-light, or lamp-light, brought to a focus at the point of incidence, should be employed.

292. Fresnel made the very striking discovery that glass, and other simply refracting bodies, are rendered doubly refracting when in a state of strain. To this Brewster added the observation that the requisite strain might be produced by unequal heating instead of by mechanical stress, and also that unannealed glass is

usually doubly refractive. Clerk-Maxwell[1] showed that shearing stress in viscous liquids, such as Canada balsam, renders them temporarily doubly refractive. This subject has been elaborately investigated by Kundt.[2]

293. Quartz is, like Iceland spar, a uniaxal crystal, but differs in an extraordinary manner from it as regards its double refraction. The wave-surface is still, as Huygens suggested for Iceland spar, a concentric sphere and ellipsoid of revolution. But here the ellipsoid is *prolate*, and lies wholly within the sphere, not touching it even at the extremities of its axis. So far as this difference is concerned, the construction for the refracted rays presents no greater difficulty than before, only that, as is obvious, the extraordinary ray is refracted more than the ordinary ray.

But neither of the rays is plane polarised. This very remarkable discovery was made by Fresnel, who examined its consequences with great care and skill. The rays are in general elliptically polarised, and rotate in opposite directions; but the ellipses become circles when the rays are parallel to the axis of the crystal. It would take us too much into detail to examine the general question, so we will confine ourselves to rays passing perpendicularly through a plate of quartz cut perpendicular to the axis; and for further simplicity we will suppose the incident light to be homogeneous and plane polarised.

Now it is an obvious kinematical theorem that the resultant of two equal uniform circular motions of equal periods, in opposite directions, in the same plane, is simple harmonic motion. For let P, Q (fig. 47) be simultaneous positions of points, moving with

[1] *Proc. Roy. Soc.*, 1873. [2] *Pogg. Ann.*, 1879.

equal but opposite angular velocities in the circle. OA, which bisects POQ in one position, will bisect it in *every* position. Hence the (equal) resolved parts of the motions of P and Q, parallel to OA, are to be *added* for the resultant motion. Their motions perpendicular to OA are evidently equal and opposite at every instant, and when superposed destroy one another.

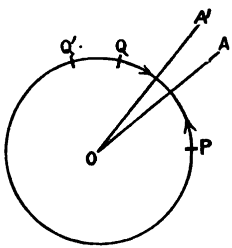

FIG. 47.

Now the position of OA depends, as we have seen, on *any* simultaneous positions of P and Q. If Q should be retarded by any cause so that P, Q′ are simultaneous positions, the resultant simple harmonic motion will be exactly the same as before, but it will be along OA′ (equally inclined to OP, OQ′) instead of OA.

But we have seen that in quartz one of the two wave-surfaces lies wholly within the other. Hence one of the rays takes a longer time to pass through the plate than does the other; and thus the portions of the two, which recombine into plane polarised light after passing through the crystal, are not relatively in the same state as when they entered it. The plane of polarisation is therefore turned through an angle which is directly proportional to the thickness of the plate.

294. When we use homogeneous light of a shorter wave-length, other things being the same, the amount of rotation is found to be greater. According to the measurements of Broch, it is (roughly) inversely as the square of the wave-length. The rotation per millimètre of quartz is about 22° for the D line of sodium. Thus, even with very small thicknesses of quartz, the rotatory *dispersion* of the planes of polarisation of the different constituents of white light is very great; for a single millimètre it is about 30° for that portion of the range of the spectrum, which can almost always be observed. As there is (practically) no additional loss of light incurred by using a thick plate of quartz instead of a thin one, it is probable that for many spectroscopic purposes this process would be almost incomparably superior to any other, provided we could obtain sufficient thicknesses of perfectly homogeneous material. Unfortunately, this condition seems very difficult of attainment.

295. When sunlight, plane polarised, is transmitted through a plate of quartz, and examined by a prism, it gives the usual spectrum. But when this spectrum in its turn is examined through a Nicol, one or more dark bands appear, parallel to the Fraunhofer lines. These bands are narrower as they are more numerous, and more numerous as the plate is thicker. They travel along the spectrum in one direction or other, as the Nicol is made to rotate positively or negatively. The cause of this appearance is obvious from the statements of § 293.

296. The amount of the rotation, of the plane of polarisation of any particular ray, is very nearly the same, for the same thickness, in plates of quartz (cut per-

pendicular to the axis) from different crystals. But the sense of the rotation is positive or right-handed in some specimens of quartz; negative or left-handed in others. Sir J. Herschel made the important observation that this peculiarity in any one specimen is always connected with the *aspect* in which certain small plagiëdral faces lie on the crystal.

297. Phenomena of essentially the same character as those just described, but not connected with any special axis, are presented by numerous liquids, such as turpentine, essential oils, solutions of sugar, etc.: and the observation of the amount of rotation of the plane of polarisation of a particular ray, by passing it through a standard thickness of such a liquid, is often a convenient substitute for chemical analysis, especially for manufacturing purposes. Various devices have been employed, called *Saccharimeters*, but they are all practically dependent on the principles above explained.

It need only be added that the rotation produced by any of these liquids is very small compared with that due to an equal thickness of quartz, and that some of them give right-handed rotation, others left-handed.

298. In any one specimen, however, of quartz or liquid, if the ray be reflected back so as to pass twice through it, the rotation is exactly *undone*. The plate of quartz, or the layer of liquid, has exactly the same properties at its two sides; just as a corkscrew (whether right or left handed) is the same at either end.

But it was one of the most important of Faraday's discoveries, that a rotation of the plane of polarisation takes place when light is transmitted through certain bodies placed in the magnetic field (1846). For simplicity we will suppose that (as in Faraday's experiments) the light passes in the direction of a line of magnetic force, *i.e.* in the direction in which a very small magnet, free to turn, would place itself. [We may remark in passing that Verdet found, by measurement, that, if the direction of the ray be inclined to the line of magnetic force, the amount of the rotation is (*cæteris paribus*) proportional to the cosine of the inclination.] The amount of the rotation depends on the nature of the body, the thickness of the plate or layer operated on, and the intensity of the magnetic field, being directly proportional to each of the latter. The direction of the rotation is, as Verdet showed, in some paramagnetic bodies opposite to that in diamagnetic bodies. The approximate law of the inverse square of the wave-length (§ 294) is also found to hold; but here the resemblance ends. For, if the ray be reflected back through the medium under magnetic force, the amount of rotation is found to be *doubled*. Hence the medium has now opposite properties at its two sides; they differ, in fact, as the two poles of a rotating body like the earth differ from one another. Along the axis of a crystal of quartz there is dipolar symmetry, along the lines of force in a transparent diamagnetic there is dipolar asymmetry.

299. It would appear, as was first pointed out by Thomson, and as follows from Clerk-Maxwell's theory of the electro-magnetic field, that there must be rotation in the luminiferous medium within the body, everywhere round the lines of magnetic force; and that this is the direct cause of the phenomenon. Thus, as the rotation of the plane of vibration, in Foucault's pendulum experiment, may be explained by the fact that (relatively to

the earth's surface) the time of a complete rotation of a conical pendulum differs according as it rotates with, or against, the earth's rotation about its axis, so in the two circularly polarised rays into which a plane polarised ray is divided in the magnetic field, the *periods* are different. In one of the rays it is less, in the other greater, than that of the incident light. Thus the difference of phase, required for the explanation of the electro-magnetic rotation of the plane of polarisation, may depend merely upon greater speed of rotation in one of the component circularly polarised rays, and not upon difference of rate of propagation as in quartz (§ 293). But experimental determinations on this very interesting point are wanting.

It has been quite recently shown by Röntgen, and others, that the Faraday effect is given by gases, and that it may be discovered, in sufficient thicknesses of gas, even when the earth's magnetism is the sole cause.

300. This is not the place to discuss electric theories, so we must content ourselves with the remark that, according to Clerk-Maxwell's theory,[1] the speed of propagation of transverse waves, in the medium which is required for the explanation of electric and magnetic phenomena, is the ratio of the electro-magnetic and the electro-static units of electricity. This is a quantity which can be measured by processes purely electrical. And it is found that the various values of this quantity, given by different experimenters, agree with those of the speed of light, at least as closely as the several determinations of the value of either agree among themselves. Hence we see that, possibly in the immediate future,

[1] See his *Electricity and Magnetism*, Chap. XX.

the subjects of light and electricity will have to be studied together: not merely because both depend upon energy, but because both depend upon energy associated with one and the same medium—a medium which is certainly material, though it has as yet eluded the processes and resources of the chemists. The reader is recommended to peruse, carefully, the article "Ether," by Clerk-Maxwell, in the latest edition of the *Encyclopædia Britannica.* He will find there, in a brief but very suggestive form, an account of all that is yet known about the properties and functions of this extraordinary substance.

CHAPTER XVI.

RADIATION AND SPECTRUM ANALYSIS.

301. IN the preceding chapters we have treated of light in its progress through various media to the eye; and we have dealt with it:—(a) as propagated according to definite, though in certain cases only approximate laws; (β) as due to a species of wave-motion: but we have as yet said nothing as to the conditions under which it is given off by a luminous source, or as to the connection between the radiating and absorbing powers of a body. This part of our subject is of comparatively recent origin, and can be treated by itself.

But we must now greatly expand our ideas, as in § 3 above, and look upon luminous rays as merely that particular class of radiations each of which is capable of affecting our eyes. In order to study their properties from the modern point of view, as forms of energy, it is necessary to consider *all* classes of ethereal radiations together. Here we must digress into matters which are usually treated in connection with *Heat*.

302. First we must give, as briefly as possible, the reasons for assuming that all radiation in the luminiferous medium is of essentially the same character, and that

what we call light is merely that portion which is capable of affecting our sense of sight.

We find a perfectly analogous phenomenon in the case of waves in air. There can be no doubt from the known properties of air, that it is capable of propagating, and does propagate, waves very much longer or very much shorter than those of any audible sound, though of precisely the same dynamical character. We do not call them sounds, simply because we do not *hear* them. The range of audible sounds differs considerably in different individuals. Some are painfully affected by the sounds from the longer pedal pipes of an organ, some by those of very short *grilli* pipes; while the great majority cannot hear any sound from either.

When we gradually heat a platinum wire, by passing an electric current through it, it begins by radiating heat only; presently it radiates red light, along with an increased amount of dark heat. As the temperature further rises, all the lower radiations increase in intensity, and new and higher ones come in, till at last the wire is white-hot—*i.e.* it gives off all kinds of visible light, in much the same proportion as that in which we receive them from the sun. When the current is interrupted, the same succession of phenomena occurs, in the opposite order, as the wire cools. One point to be particularly observed, as it is extremely important to the theory of spectrum analysis, is that *each special kind of radiation increases in intensity as the temperature rises.*

303. In homogeneous media, all kinds of radiation are propagated in straight lines. Hence the heating, like the illuminating power of a source, is inversely as the square of the distance.

They seem all to be propagated, in free space, with the remarkably great velocity of light. At least, the sun's heat is intercepted by the moon, in a total eclipse, at the instant that the last trace of the disk vanishes.

They are all reflected according to the same law. A burning mirror is adjusted by means of the luminous rays.

Heat rays are less refracted than red rays, just as red rays are less refracted than blue, by any one medium. We might go through a whole series of other analogies, but the two which follow seem absolutely decisive of the identity.

Fizeau and Foucault proved that the ordinary diffraction experiments succeed with dark heat as with light, only indicating a greater wave-length.

Forbes conclusively established the polarisation of dark heat, and its double refraction.

304. Prevost, in the end of last century, first distinctly enunciated the statement that the radiation from a body depends, for its quality and quantity, upon the body itself, and its temperature, *alone*. Thus the equality of temperature which is ultimately attained, as experiment shows, by all bodies contained in an enclosure impervious to heat and containing no source of heat, is maintained by a constant *exchange* of heat between the members of each pair of the bodies. And the attainment of equality of temperature depends on the fact that the amount of radiation from any one body increases as its temperature is raised. For, when there are initial differences of temperature, the hotter bodies radiate more and receive less, and the colder radiate less and receive more, than when the final state has been

arrived at. This is true, but it is by no means the whole truth. De la Provostaye was the first to throw additional light on the subject; but, though he advanced in the right direction, the first approximately complete statement of the matter, as we now know it, was given by Balfour Stewart in 1858.[1] The main features of his work are not very hard to follow, and will be given presently. But some preliminary statements and definitions are indispensable. We will make them as brief as possible.

305. It was known, mainly by the valuable experiments of Leslie at the very beginning of the century, how very greatly bodies differ in their radiating powers, even when all are at the same temperature. Also that the better radiating bodies are in general those, such as lamp-black, which absorb nearly all the light which falls upon them; while polished silver, which is an excellent reflector, radiates only about $\frac{1}{40}$th as much as lamp-black under similar conditions. De la Provostaye and Desains extended the inquiry, with the result of showing that the radiating and absorbing powers of one and the same body are proportioned to one another, and become less as the reflective power of the body is increased—as, for instance, by altering the texture of its surface. But all these measurements referred to the total radiation, and no attempt seems to have been made to confine them to small specific classes of rays.

306. But while this was the case with radiant heat, it was otherwise with light. The first suggestion as to the origin of the dark lines in the solar spectrum (§ 139) seems to have been made by Brewster, who was led to

[1] *Trans. R. S. E.*, 1858. On an extension of Prevost's Law. See also *Phil. Mag.*, 1863. I. 354.

it by his discovery of the remarkable series of absorption-bands seen in the spectrum of light which had passed through peroxide of nitrogen.[1] Brewster proceeded, by comparing the spectrum of sunlight at mid-day with that at sunset, to show that some at least of the lines in the spectrum are due to the earth's atmosphere.

An exceedingly close approach to the full explanation was made in 1849 by Foucault,[2] who found that, while the electric arc gave two *bright* lines in the place of Fraunhofer's double line D (§§ 136, 139), the light from one of the carbon points (which of itself gives a continuous spectrum) showed the D lines *dark* when it was passed by reflection through the arc.

About 1850 Stokes explained the phenomenon by an analogy drawn from sound. He pointed out that a space filled with stretched wires, or tuning-forks, all set to one pitch, would (when these were in vibration) give off that particular note alone; but that, if it were interposed between a performer on a powerful instrument and an audience, it would specially absorb and weaken that particular note.

Ångström, in 1853, stated, as the result of numerous experiments, that the rays which a gas can absorb are precisely those which it can give off when luminous.

No further action, however, seems to have been taken in consequence of these very definite statements and analogies. It became again the turn of radiant heat, and from its behaviour Stewart, in 1858, published his extension of Prevost's theory (§ 304).

307. Just as dynamical reasoning is simplified by the

[1] *Trans. R. S. E.*, 1836. *Phil. Mag.*, 1836. I. 384.

[2] *L'Institut*, Feb. 7, 1849. See *Phil. Mag.*, 1860. I. 193.

introduction of ideal properties never found realised in ordinary matter, such as those of *rigid* bodies, *perfect* fluids, etc., so we may simplify the present investigation by the following abstract conceptions:—

1. A *black* body is one which absorbs every ray which falls on it. It can, therefore, neither reflect nor transmit. A mass of coke suggests the conception of such a body.

2. A *perfectly reflecting* body is one which cannot absorb any ray. Polished silver suggests such a body.

3. Coloured glasses of various kinds (especially those coloured by didymium) suggest the conception of a body *perfectly absorbent* of one or more definite kinds of radiation, and transparent to all others. We may make this conception less restricted by assuming the absorption of the particular ray to be *partial* only.

Many more such conceptions are suggested by the results of experiments on the properties of various materials, but those just given are sufficient for our immediate purpose.

308. Let a number of bodies of any kind be enclosed in a perfectly reflecting envelope, within which there is no source of heat, then by our fundamental experimental fact (§ 304) they will ultimately reach one common temperature and retain it unaltered.

This equilibrium of temperature is maintained by constant radiation, reflection, and absorption, which (for the moment) we assume to take place at the surface of each one of the bodies. Hence at every point within the enclosure the radiation which passes (either way) perpendicularly across unit of surface, oriented in any way, must be the same. For any number of additional bodies, if at the proper temperature, may be introduced without affecting the equilibrium.

Also, as one of the bodies may be a black body, we see that this radiation must be that of a black body at the temperature of the contents of the enclosure.

309. Place in front of the black body a body of the class (3) of § 307 also at the proper temperature. This cannot, as we have seen, alter the nature of the radiation. But it does abstract from the radiation of the black body the whole or part of one particular ray. Hence it must make up for this by emitting that ray to exactly the same amount as it absorbs it.

The *Absorptive Power*, under any circumstances, for a particular radiation, is defined as the ratio which the part absorbed bears to the whole incident radiation of that kind.

Thus, if we now define the *Emissivity* of a body at a given temperature, for a particular radiation, as the ratio of its emission of that radiation to the emission of the same radiation by a black body at the same temperature, we see that *the emissivity of a body for any radiation is equal to its absorptive power for the same radiation at any one temperature.*

310. The experiments by which Stewart tested these results were, for the greater part, confined to heat rays. But he gave a very striking illustration of the theory, as regards light, from the behaviour of coloured glass put into a clear fire. For a red glass appears red so long as it is colder than the coals behind it, apparently loses all colour when it is at the same temperature as these, and shows the complementary colour (green) when a colder coal is behind it.

311. Kirchhoff's investigations[1] on the same subject appeared a year or so later, but were more immediately connected with light. His proof of the main proposition (§ 309) is essentially based on the same experimental result as that from which Stewart proceeded; but he employed mathematical methods, instead of simple conceptions like (3) of § 307, to limit the reasoning to one definite kind of radiation. Thus the investigation is by no means easy reading for the ordinary student.

But Kirchhoff's experimental verifications of his result were almost exclusively confined to luminous rays. One of these is specially important. When chloride of lithium is placed in the flame of a Bunsen lamp, the flame assumes a dark crimson colour, which is found to be due to radiation of one definite wave-length. Sunlight shows no deficiency in this particular ray, but it does so under certain conditions, when passed through the lithium flame.

312. Another extremely beautiful verification of the theory, at which Kirchhoff and Stewart independently arrived, is this:—that the radiation from a heated plate of tourmaline, cut parallel to the axis of the crystal, is polarised. The plane of polarisation is found to be perpendicular to that of light which has passed through the plate (§ 279); it is therefore coincident with that of the light absorbed by the plate.

313. Kirchhoff also gave, but in a much more precise form, the result of Stewart which was described in § 310.

He found experimentally that, when the source of light is an ignited lime-ball (the Drummond Light),

[1] Berlin Acad., *Monatsbericht*, Oct. 1859. Also *Pogg. Ann.* cix. 275.

which gives a continuous spectrum, *bright* D lines are produced by passing the light through a Bunsen flame containing common salt, but *dark* D lines when the colder flame of a spirit lamp is used.

The theory of this is obvious from the general result of § 309. For, if E be the intensity of a particular radiation from the source, E′ that of the same radiation from a black body at the temperature of the flame, and e the emissivity (or absorbing power) of the flame itself for the same radiation, the intensity of that radiation, after it has passed through the flame, is obviously

$$E(1-e) + eE'.$$

The first term is what the flame allows to pass, the second is what it supplies. The expression may be put in the form

$$E + e(E' - E).$$

Hence there is strengthening or weakening by the flame, as E′ is greater or less than E. Thus, to produce a dark line in an otherwise continuous spectrum, the absorbing body must be at a temperature so low that a black body at that temperature will not emit the particular radiation concerned so powerfully as does the source.

It is interesting to study the behaviour of a lithium flame with sunlight, as the sunlight is gradually weakened by the introduction of a wedge of neutral tinted glass. This has the effect of diminishing E in the formula above, so that the dark line which is produced by the flame in full sunlight is gradually weakened, then disappears, to be succeeded by a bright line which gradually becomes more marked.

314. It will be observed that the essential basis of

the reasoning, by which the above results were arrived at, is the ultimate equality of temperature among a number of bodies in an enclosure impervious to radiation. This is undoubtedly true, as a general experimental fact, but it is so *only* in consequence of the practically infinite size of a thermometer bulb in comparison with the size of the particles of matter. The kinetic theory of gases shows us that, in a gas at uniform temperature, as it is called, the majority of the particles are moving with speeds not very different from the *velocity of mean square*, but that the remainder have speeds greater or less in every possible ratio. The number having any particular speed no doubt becomes rapidly smaller, the more that speed differs from the mean. But the phrase "uniform temperature" is an expression to be justified only in a statistical sense, as the average of irregularities too regularly spread, and on a scale too small, to be detected by our instruments. Hence the whole theory, which really involves Carnot's Principle (the basis of the *Second Law of Thermo-dynamics*), is true only in the same sense as Carnot's Principle itself is true.

We have only to think of luminous paint (§ 204) to see that there are grave exceptions to the assumption of § 302 that all bodies require to be raised above a certain (high) temperature before they can radiate visible light. Thus it is absurd to speak, as many authors have recently done, of a *rigorous* proof of the equality of absorption and emissivity. The kernel, at least, of the proper treatment of this question is to be found in the remarks of Stokes about fluorescence (§ 201); but in this elementary book we content ourselves with the reasoning already given, allowing that its grounds are approximate only, but claiming that the results are, except in special cases of great difficulty, found to be realised in practice.

315. The mode in which Stewart originally gave the statement of § 309 brought more explicitly forward than we have done the fact that radiation is not confined to the *surface* of a partially transparent body, but takes place from all the internal particles as well. But it is quite clear that the thickness of the absorbing plate there spoken of is immaterial to the result, and therefore that the radiation through it is strengthened by fresh internal radiation, exactly to the amount of the weakening by absorption, in every stage of its progress through the plate. We thus see how the results of § 309 are at once deducible from the theory.

Stewart experimentally illustrated this additional point by proving that a thick plate, of any partially transparent material, radiates more at the same temperature than a thin one. He also showed that the internal radiation, so long as the body is homogeneous and isotropic, must be proportional to the *square* of the refractive index. This is easily seen by considering how a small pencil of rays is widened by refraction as it emerges from the body into air.

316. We can spare but a few paragraphs to the processes and results of *Spectrum Analysis*, of which we have just given the approximate theoretical basis. The subject has been so extensively developed within the last twenty years that there are now many excellent special treatises (in different languages) wholly confined to its principles, practice, and teachings.

317. In §§ 130, 141, 155, we showed how to procure a pure spectrum by means of a slit, a *thin* prism, and an

achromatic lens. An arrangement of one or more prisms of large angle, provided each be placed in the position of *minimum deviation* (§ 134), improves the results by greatly increasing the whole dispersion. To obtain the best effect, however, in this case, it is necessary that the beam of light should consist of *parallel* rays before incidence, and (therefore) after refraction. This is accomplished by placing an achromatic lens in front of the prism, and at its focal distance from the slit. This arrangment is called a *collimator*. Rays diverging from the slit are rendered parallel by the collimator, and an achromatic telescope (focussed for parallel rays) is employed to examine them after refraction.

In place of the prism, or train of prisms, we may of course employ a grating (§ 235).

318. By any one of these processes we obtain a pure spectrum of the light from the source employed. This consists of a series of images of the slit, parallel to one another, each due to one of the constituents of the light, and is therefore continuous if the radiation is so. Any two of these images, due to definite wave-lengths, are separated from one another by an angular interval depending upon the dispersion and upon the magnifying power employed.

a. If the source be a black body (§ 307) the spectrum is continuous, for such a body radiates (as it absorbs) every wave-length. The brightness and also the upper limit of this spectrum depend upon the temperature of the body. Conversely, when the spectrum is continuous, the source is something which behaves, at least approximately, like a black body. This may be a partially transparent solid or liquid, or even a gas; but in the latter case it must be either of very great thickness or very great density.

β. But if the source be an incandescent gas or vapour, in not too great thickness and not under extreme pressure, the spectrum usually consists of a series of isolated images of the slit. Even if the radiation corresponding to each of these images were absolutely homogeneous, and the slit of infinitesimal breadth, the corresponding image must be of finite (though small) breadth, because the various particles of a hot gas are moving at different speeds to or from the spectator, and the Döppler principle (§ 254) comes in. Also, during the collisions of the particles, their vibrations are constrained, and the wave-length of the emitted light is altered. The denser the gas, *i.e.* the greater proportion the time of collision bears to the time of free path, the more this tells; and when the gas is compressed to nearly the density of the corresponding liquid, the separate images have broadened out so as to meet one another, and we have the case treated under (*a*) above.

γ. If the gas, considered as a whole, have rapid internal currents, the Döppler principle again comes in, and the lines are broadened (sometimes bodily shifted) towards the less refrangible side by the radiation from the parts receding from the spectator, and *vice versâ*.

δ. As each particle of a gas is subjected to nearly periodic impacts by its fellow particles, or those of another gas mixed with them, we may expect to find that the *relative* intensities of the various rays which it can give out will depend on the temperature, and the amount of admixture: just as a bell, which has an infinite number of special periods of vibration, will give out different characters of mixed sounds according to the special period of a persistent exciting cause of its vibrations.

The general principle of § 309 shows that the same remarks will apply if the bodies be employed to absorb light (from a black body) instead of radiating it.

319. The student may verify these results in an exceedingly simple manner by the help of a single good prism held close to the eye (as in Wollaston's original experiment, § 139), with a glass grating (§ 235), or better, with a so-called *direct-vision* spectroscope.

When the source of light is a very thin wire, heated in a Bunsen flame, and the edge of the prism or the lines of the grating are placed parallel to it, a slit is not necessary: and a continuous spectrum is observed, whose general brightness and whose extension towards the violet both depend upon the temperature of the wire (§ 302).

The flame itself (limited by a narrow slit) gives the peculiar spectrum of the Hydrocarbons, consisting of three bright bands, each sharply terminated towards the red end, and gradually becoming fainter towards the violet. This spectrum had been carefully studied by Swan, before the publication of the reasonings of Stewart and Kirchhoff. Huggins and others have found it to be characteristic of the intrinsic light of several comets.

320. Now introduce into the flame a wire dipped in brine, or in a strong solution of chloride of lithium, calcium, barium, magnesium, thallium, etc., and we have at once the characteristic bright line spectrum of the corresponding metal. The chlorides are chosen because they are found to be the most volatile, and also the most easily decomposed at the temperature of the Bunsen flame.

With a more powerful spectroscope, adjusted to a divided circle, the refractive index for each of these rays can be accurately ascertained.

321. For the verification of the *reversal* of spectral lines, the process of Kirchhoff (§ 311) may be used. Here the source (preferably a black body, but a Drummond lime-ball will do) must usually be at a considerably higher temperature than the absorbing flame—so that a common spirit-lamp flame should be employed.

322. For the less volatile metals, such as iron, titanium, nickel, etc., the electric arc, or the spark from a powerful induction coil, is required. With the former source of heat, a small fragment of the metal is placed on the lower carbon electrode; with the latter, the electrodes themselves are usually wires of the metal to be studied.

323. The most striking exhibition of reversal is that given, in imitation of Foucault's original experiment (§ 306), by passing the continuous radiation from one of the incandescent carbon terminals of the electric arc through a train of prisms and a lens, so as to give a continuous spectrum on a screen (§ 155), and then interposing (between the source and the prisms) a Bunsen flame in which a piece of metallic sodium is heated to incandescence. [An opaque body should be placed so as to prevent the *direct* light of the burning sodium from falling on the screen.]

Another curious form of this experiment is to place a small spirit-lamp flame, with a wire dipped in brine inserted near its base, in front of a large and powerful Bunsen flame containing a pellet of metallic sodium. This acts as a back-ground; and upon it, as Bunsen showed, the colder flame appears dark, like a piece of crape.

324. The first practical application of this theory was to the classification of the spectra given by different elementary bodies when incandescent; and the almost immediate result was the detection of two new metals (cæsium and rubidium) in the concentrated brine procured by evaporating large quantities of a mineral water from Dürckheim. Their presence was proved by the existence of bright lines, not traceable to any known element. Bunsen was thus led to isolate these metals by appropriate chemical processes.

Kirchhoff, on the other hand, undertook the laborious work of studying the dark lines of the solar spectrum, and comparing their refrangibility with that of the bright lines of the spectra of incandescent terrestrial substances. He thus arrived at the conclusion that a very large number of the Fraunhofer lines are due to metallic iron, in a vaporous form, in the sun's atmosphere. This particular inquiry has since been developed by numerous investigators, among whom may be particularly mentioned Ångström and Thalén, Cornu, and Piazzi Smyth.

The diagram, fig. 48, which is taken from Ångström's great work (*Spectre normal du Soleil*) shows a small portion of the solar spectrum (in the green region), including the three magnesium lines called by Fraunhofer *b*. The numbers above it are the wave-lengths in millionths of a millimètre (§ 241), and the companion figures below show the positions of the bright lines of metallic iron (in a spectrum by themselves), and of other terrestrial elements, corresponding to the same range of wave-lengths. The coincidences are obvious from the figure, and so are the corresponding conclusions as to the constitution of the sun's atmosphere.

325. So far, we have spoken of sunlight as a *whole*. But when, by means of a lens of long focus, we produce an image of the sun, of an inch or two in diameter, we can place the slit of the spectroscope at any desired point of the image, and thus study the *special* radiation from the corresponding region of the sun. When this is done, as it has been by Janssen, Lockyer, and many others, the comparative simplicity and uniformity observed in the spectrum of the total sunlight are found to give place

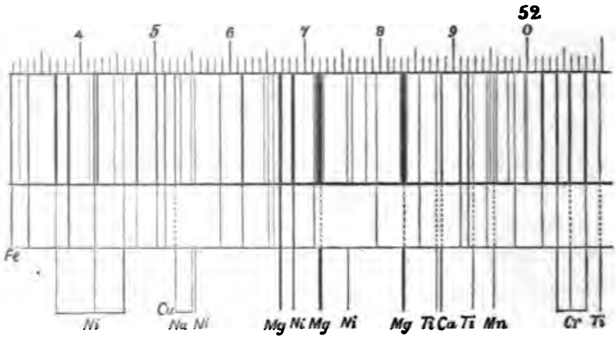

FIG. 48.

to an ever-changing variety of appearances, some of a very puzzling character. This is especially the case in, and in the neighbourhood of, spots and *faculæ*. Sometimes the hydrogen lines, such as F, are seen bright, on a comparatively dark ground; sometimes they are crooked, indicating (§ 318, γ) great local differential motion. Again one or more of the lines due to one element are found bright, while others are dark; or crooked, while the others are straight, etc. We cannot examine such cases in detail, but the statement (δ) of

§ 318 gives an idea of the difficulty of the question; which is still further complicated by the fact that what we observe at any point of the sun's surface is the "integral," as it were, of all the radiation and absorption which take place throughout a column of violently heated and rapidly moving gases somewhere about 100,000 miles in height, with the luminous body of the sun behind them. The distribution of temperature and velocity throughout this column may be of the most varied character.

326. A partial clue to the solution of this problem is afforded by the study of the spectrum of the solar atmosphere outside the apparent edge of the disc. Observations of total eclipses of the sun had, long ago, shown that certain "red protuberances," as they were called, were seen round the dark body of the moon. That these belonged to the sun was conclusively proved by observations and photographs taken during the great eclipse of 1860. To Lockyer and Janssen, independently, we owe the discovery that it is possible to examine these protuberances even when there is no eclipse. Their light is found to be due mainly to incandescent hydrogen, *i.e.* it consists of a few definite wave-lengths only. These *retain their brightness*, however great be the dispersion of the train of prisms employed, while the rest of the light (mainly the glare from the earth's atmosphere), giving a nearly continuous spectrum, is weakened in proportion as the dispersion is greater. [The principle involved is exactly the same as that which leads to telescopic observations of stars by daylight.] The slit of the spectroscope may now be widened, and thus we have a set of pictures of the protuberance, each formed by one of the

homogeneous rays which alone it emits. But the pictures thus given may not, by any means, accurately represent the *form* of the object. For we are looking at a representation which may be very materially altered (by the Döppler principle) in consequence of the extraordinary speed with which, in many cases, these solar storms have been shown to move.

327. The particularly well-marked lines A and B (see § 241) were assigned by Ångström to the earth's atmosphere. Piazzi Smyth greatly strengthened the proof by careful observations made on the solar spectrum, at sunrise and at mid-day, in the clear atmosphere of Lisbon; and quite recently Egoroff has directly proved, by using long tubes filled with compressed oxygen, that they are due to that gas. Special lines, and cloudy bands, have been found (some, long ago, by Brewster) to be dependent on the amount of aqueous vapour in the air. One of these, which is close to, but on the less refrangible side of, the D line, has been characterised by Smyth as the *Rain-band*, and is found to be of high import in the matter of weather forecasts.

328. We cannot further dilate on these matters, which have already had whole volumes devoted to them. We may merely mention that, when the methods above spoken of are applied to stellar light, the results obtained are necessarily of the class described in § 324, where we spoke of sunlight as a whole. But the constituents indicated by the absorption lines vary very much from star to star. In this way four very definite *classes* of stellar spectra have been recognised, the physical import of which has not yet been fully traced. It is, at least, a plausible idea that we can thus gather informa-

tion as to the relative *ages* of various stars; those which are merely nascent, as well as those which are nearly extinct, giving on the whole faint continuous spectra crossed by a few bright lines; while those which are at their hottest show a few lines only, and these usually broad as well as dark ones. Such is the case with Sirius, Vega, etc., which are *blue* stars. Our own sun belongs to the class of *yellow* stars, probably somewhat *past* maturity, and the spectra of this class are crossed by a multitude of fine lines. Others, still more aged, show fluted bands instead of sharp lines as the characteristic of their spectra. The occasional outburst of a star, in which its brightness suddenly increases to a notable amount, has occurred in one well-marked instance since the new methods came into general use. This was found to be due to a mass of incandescent hydrogen, probably analogous to the red protuberances so constantly observed in our own sun.

APPENDIX.

I.—HAMILTON ON THEORIES OF LIGHT (§ 181).

"Those who have meditated on the beauty and utility, in theoretical mechanics, of the general method of Lagrange—who have felt the power and dignity of that central dynamical theorem which he deduced, in the *Mécanique Analytique* . . . —must feel that mathematical optics can only *then* attain a co-ordinate rank with mathematical mechanics . . . , when it shall possess an appropriate method, and become the unfolding of a central idea. . . . It appears that if a general method in deductive optics can be attained at all, it must flow from some law or principle, itself of the highest generality, and among the highest results of induction. . . . [This] must be the principle, or law, called usually the Law of Least Action; suggested by questionable views, but established on the widest induction, and embracing every known combination of media, and every straight, or bent, or curved line, ordinary or extraordinary, along which light (whatever light may be) extends its influence successively in space and time: namely, that this linear path of light, from one point to another, is always found to be such, that if it be compared with the other infinitely various lines by which in thought and in geometry the same two points might be connected, a certain integral or sum, called often *Action*, and depending by fixed rules on the length, and shape, and position of the path, and on the media which are traversed by it, is less than all the similar integrals for the other neighbouring lines, or, at least, possesses, with respect to them, a certain *stationary* property. From this Law, then, which may, perhaps, be named the LAW OF STATIONARY ACTION, it seems that we may most fitly and with best hope set out, in the synthetic or deductive process, and in the search of a mathematical method.

"Accordingly, from this known law of least or stationary action, I deduced (long since) another connected and coextensive principle, which may be called, by analogy, the LAW OF VARYING ACTION, and which seems to offer naturally a method such as we are seeking: the one law being as it were the last step in the ascending scale of induction, respecting linear paths of light, while the other law may usefully be made the first in the descending and deductive way.

"The former of these two laws was discovered in the following manner. The elementary principle of straight rays showed that light, under the most simple and usual circumstances, employs the direct, and, therefore, the shortest course to pass from one point to another. Again, it was a very early discovery (attributed by Laplace to Ptolemy) that in the case of a plane mirror, the bent line formed by the incident and reflected rays is shorter than any other bent line, having the same extremities, and having its point of bending on the mirror. These facts were thought by some to be instances and results of the simplicity and economy of nature; and Fermat, whose researches on maxima and minima are claimed by the continental mathematicians as the germ of the differential calculus, sought anxiously to trace some similar economy in the more complex case of refraction. He believed that by a metaphysical or cosmological necessity, arising from the simplicity of the universe, light always takes the course which it can traverse in the shortest time. To reconcile this metaphysical opinion with the law of refraction, discovered experimentally by Snellius, Fermat was led to suppose that the two lengths, or *indices*, which Snellius had measured on the incident ray prolonged and on the refracted ray, and had observed to have one common projection on a refracting plane, are inversely proportional to the two successive velocities of the light before and after refraction, and therefore that the velocity of light is diminished on entering those denser media in which it is observed to approach the perpendicular: for Fermat believed that the time of propagation of light along a line bent by refraction was represented by the sum of the two products, of the incident portion multiplied by the index of the first medium, and of the refracted portion multiplied by the index of the second medium; because he found, by his mathematical method, that this sum was less, in the case of a plane refractor, than if light went by any other than its actual path from one given point to another; and because he perceived that the supposition of a velocity inversely as the index, reconciled his mathematical discovery of the minimum of the foregoing sum with his cosmological principle of least time. Descartes attacked Fermat's opinions respecting light, but Leibnitz zealously defended them; and Huygens was led, by reasonings of a very different kind, to adopt Fermat's conclusions of a velocity inversely as the index, and of a *minimum time* of propagation of light, in passing from one given point to another through an ordinary refracting plane. Newton, however, by his theory of emission and attraction, was led to conclude that the velocity of light was *directly*, not *inversely*, as the index, and that it was *increased* instead of being *diminished* on entering a denser medium; a result incompatible with the theorem of shortest time in refraction. This theorem of shortest time was accordingly abandoned by many, and among the rest by Maupertuis, who, however, proposed in its stead, as a new cosmological principle, that celebrated *law of least action* which has since acquired so high a rank in mathematical physics, by the improvements of Euler and Lagrange.

"Maupertuis gave the name of *action* to the product of space and velocity, or rather to the sum of all such products for the various elements of any motion; conceiving that the more space has been traversed and the less time it has been traversed in, the more action may be considered to have been expended: and by combining this idea of action with Newton's estimate of the velocity of light as increased by a denser medium, and as proportional to the refracting index, and with Fermat's mathematical theorem of the minimum sum of the products of paths and indices in ordinary refraction at a plane, he concluded that the course chosen by light corresponded always to the *least possible action*, though not always to the least possible time. He proposed this view as reconciling physical and metaphysical principles, which the results of Newton had seemed to put in opposition to each other; and he soon proceeded to extend his law of least action to the phenomena of the shock of bodies. Euler, attached to Maupertuis, and pleased with these novel results, employed his own great mathematical powers to prove that the law of least action extends to all the curves described by points under the influence of central forces; or, to speak more precisely, that if any such curve be compared with any other curve between the same extremities, which differs from it indefinitely little in shape and in position, and may be imagined to be described by a neighbouring point with the same law of velocity, and if we give the name of *action* to the integral of the product of the velocity and element of a curve, the difference of the two neighbouring values of this action will be indefinitely less than the greatest linear distance (itself indefinitely small) between the two near curves; a theorem which I think may be advantageously expressed by saying that the action is *stationary.* Lagrange extended this theorem of Euler to the motion of a system of points or bodies which act in any manner on each other; the action being in this case the sum of the masses by the foregoing integrals.

"Laplace has also extended the use of the principle in optics, by applying it to the refraction of crystals; and has pointed out an analogous principle in mechanics, for all imaginable connections between force and velocity. But although the law of least action has thus attained a rank among the highest theorems of physics, yet its pretensions to a cosmological necessity, on the ground of economy in the universe, are now generally rejected. And the rejection appears just, for this, among other reasons, that the quantity pretended to be economised is in fact often lavishly expended. In optics, for example, though the sum of the incident and reflected portions of the path of light, in a single ordinary reflection at a plane, is always the shortest of any, yet in reflection at a curved mirror this economy is often violated. If an eye be placed in the interior, but not at the centre, of a reflecting hollow sphere, it may see itself reflected in two opposite points, of which one indeed is the nearest to it, but the other on the contrary is the farthest; so that of the two different paths of light, corresponding to these two opposite points, the one indeed is the shortest, but the other is the longest of any. In mathematical language, the integral called action, instead of being always a minimum, is often a maximum; and often it is neither the one nor the other: though it has always a certain *stationary* property, of a kind which has been already alluded to, and which will soon be more fully explained. We cannot, therefore, suppose the economy of this quantity to have been designed in the divine idea of the universe: though a simplicity of some high kind may be believed to be included in the idea. And though we may retain the name of *action* to denote the stationary integral to which it has become appropriated—which we may do without adopting either the metaphysical or (in optics) the physical opinions that first suggested the name—yet we ought not (I think) to retain the epithet *least*, but rather to adopt the alteration proposed above, and to speak in mechanics and in optics of the *Law of Stationary Action*."

II.—HUYGENS ON RAYS (§ 208).

"Pour venir aux proprietez de la lumiere; remarquons premierement que chaque partie d'onde doit s'étendre en sorte, que les extremitez soient tousjours comprises entre les mesmes lignes droites tirées du point lumineux. Ainsi la partie de onde BG, ayant le point lumineux A pour centre, s'étendra en l'arc CE, terminé

par les droites ABC, AGE. Car bien que les ondes particulieres, produites par les particules que comprend l'espace CAE, se repandent aussi hors de cet espace, toutesfois elles ne concourent point en mesme instant, à composer ensemble une onde qui termine le mouvement, que precisement dans la circonference CE, qui est leur tangente commune.

"Et d'icy l'on voit la raison pourquoy la lumiere, à moins que ses rayons ne soient reflechis ou rompus, ne se répand que par des lignes droites, en sorte qu'elle n'éclaire aucun objet que quand le chemin depuis sa source jusqu'a cet objet est ouvert suivant de telles lignes. Car si, par exemple, il y avoit une ouverture BG, bornée par des corps opaques BH, GI; l'onde de lumiere qui sort du point A sera tousjours terminée par les droites AC, AE, comme il vient d'estre demonstré: les parties des ondes particulieres, qui s'étendent hors de l'espace ACE, estant trop foibles pour y produire de la lumiere.

"Or quelque petite que nous fassions l'ouverture BG, la raison est tousjours la mesme pour y faire passer la lumiere entre des lignes droites; parce que cette ouverture est tousjours assez grande pour contenir un grand nombre de particules de la matiere etherée, qui sont d'une petitesse inconcevable; de sorte qu'il paroit que chaque petite partie d'onde s'avance necessairement suivant la ligne droite qui vient du point luisant. Et c'est ainsi que l'on peut prendre des rayons de lumiere comme si c'estoient des lignes droites.

"Il paroit au reste, par ce qui à esté remarqué touchant la foiblesse des ondes particulieres, qu'il n'est pas necessaire que toutes les particules de l'Ether soient égales entre elles, quoique l'égalité soit plus propre à la propagation du mouvement. Car il est vray que l'inégalité fera qu'une particule, en poussant une autre plus grande, fasse effort pour reculer avec une partie de son mouvement, mais il ne s'engendrera de cela que quelques ondes particulieres en arriere vers le point lumineux, incapables de faire de la lumiere: & non pas d'onde composée de plusieurs, comme estoit CE.

"Une autre, et des plus merveilleuses proprietez de la lumiere est que, quand il en vient de divers costez, ou mesme d'opposez, elles font leur effet l'une à travers l'autre sans aucun empéchement. D'ou vient aussi que par une mesme ouverture plusieurs spectateurs peuvent voir tout à la fois des objets differens, & que deux personnes se voyent en mesme instant les yeux l'un de l'autre. Or suivant ce qui a esté expliqué de l'action de la lumiere, et comment ses ondes ne se détruisent point, ny ne s'interrompent les unes les autres quand elles se croisent, ces effets que je viens de dire sont aisez à concevoir. Qui ne le sont nullement à mon avis selon l'opinion de Des-Cartes, qui fait consister la lumiere dans une pression continuelle, qui ne fait que tendre au mouvement. Car cette pression ne pouvant agir tout à la fois des deux costez opposez, contre des corps qui n'ont aucune inclination à s'approcher; il est impossible de comprendre ce que je viens de dire de deux personnes qui se voyent les yeux mutuellement, ni comment deux flambeaux se puissent éclairer l'un l'autre."

III.—LAPLACE ON THE UNDULATORY THEORY (§ 219).

"J'ai reçu la lettre que vous m'avez fait l'honneur de m'écrire, et dans laquelle vous cherchez à établir que suivant le système des ondulations de la lumière, les sinus d'incidence et de réfraction sont en rapport constant, lorsqu'elle passe d'un milieu dans un autre. Quelque ingénieux que soit ce raisonnement, je ne puis le regarder que comme un aperçu, et non comme une démonstration géométrique. Je persiste à croire que le problème de la propagation des ondes, lorsqu'elles traversent différens milieux, n'a jamais été résolu, et qu'il surpasse peut-être les forces actuelles de l'analyse. Descartes expliquoit ce rapport constant, au moyen de deux suppositions; l'une, que la vîtesse des rayons lumineux parallèlement à la surface du milieu refringent ne changeoit point par la réfraction; l'autre, que sa vîtesse entière dans ce milieu étoit la même, sous toutes les incidences; mais comme il ne rattachoit aucune de ces suppositions aux lois de la mécanique, son explication a été vivement combattue et rejettée par les plus grand nombre des physiciens jusqu'à ce que Newton ait fait voir que ces suppositions résultoient de l'action du milieu refringent sur la lumière; alors on a eu une explication mathématique du phénomène dans le système de l'émission de la lumière: système qui donne, encore l'explication la plus simple du phénomène de l'aberration, que n'explique point le système des ondes lumineuses. Ainsi les suppositions de Descartes, comme plusieurs aperçus de Kepler sur le système du monde, ont été vérifiées par l'analyse: mais le mérite de la découverte d'une vérité appartient tout entier à celui qui la démontre. Je conviens que de nouveaux phénomènes de la lumière sont jusqu'à présent très difficiles à expliquer; mais en les étudiant avec un grand soin, pour découvrir les lois dont ils dépendent, ou parviendra peut-être un jour à reconnaître dans les molecules lumineuses des propriétés nouvelles qui donneront une explication mathématique de ces phénomènes. Remonter des phénomènes aux lois et des lois aux forces, est, comme vous le savez, la vraie marche des sciences naturelles."

INDEX.

NOTE.—*The figures below refer to sections, not to pages.*

THE END.

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